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IMPACT OF PANDEMIC COVID-19 ON ECONOMIC PRODUCTION - AN INVENTORY MODEL FOR DETERIORATING ITEMS

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ABSTRACT

In the current era of the pandemic covid-19, the economy of the world is shattered down due to lockdown of the system and new guidelines for businesses. In this investigation, we propose an economic production inventory model for deteriorating items with shortages and quality control. The demand rate is taken as a linear function of time and the production rate is assumed to be finite. This model is capable to deal with the shortages and backlogged fully in the system. The lead time is considered in this study to be zero, which is also advisable to fulfill the demand of the pandemic situation. Here we illustrate the entire model with the numerical findings which validate the analytical results and helpful to demonstrate the effect of system descriptors and deterioration on the total cost of the system.

Key words: Inventory; deterioration; quality control cost; price dependent demand; shortages.

1. INTRODUCTION

The main component of the production and warehouses is the stock of goods for unprdictable and future demand. The one of the impotant factors of the sucess of any business depends upon the balance of demand and supply. The economy of India during the COVID-19 pandemic has been largely affected due to lockdowns as such the demand for deteriorating items like groceries, packed food, dairy items, medicine, etc. has rapidly increased. But on the other hand, the demands in other sectors like textile industries, handicrafts, electronics, etc. have reached to its lower level. An order quantity is the number of products manufactured in one production cycle. The quality of the product plays an important role due to fact that vendors would like to ensure all the necessary steps taken to inspect quality before deliveries. The step-by-step process of reducing, improving, or eliminating manufacturing errors of products and provide the best quality products to the customers can be studied under inventory control.

The quality of products in the inventory management system is the prime concern for the profitability of manufacturers as well as the satisfaction of the clients. The profitability of the business as well as goodwill of manufacturers depends upon the availability of products as per customers' requirements. When the size of the order is large, it may reduce the frequency of orders to be placed to procure inventory items and reduce the total ordering cost (purchasing cost, cost of the inspection, etc.). But this decision will increase the cycle stock inventories and cost of carrying (cost of storage, insurance, etc.) inventories. The size of the order which reduces the annual total cost of carrying inventory and ordering inventory is Economic Order Quantity (EOQ) or Economic Lot Size (ELS). Bhandari and Sharma (1993) presented the inventory model with a variable cost function. They have used a generalized cost to provide a feasible solution to the model. Bawa and Raikhy (2001) studied the effect of production function and productivity in the industry. As the fixed capital assets are added at different points of time there is serious deficiency in the estimation of fixed capital. Recently, Basha et. al. (2020) reviewed different aspects of invetory models and provided a detailed information about different methods like ABC, VED, EOQ, JIT, etc.

In this time of large-scale production and competitive business world, the company who supplies good quality products, can survive in the system, as similar products are easily available in the market. By utilizing the scientific approaches of inventory management, the manufacturers can utilize the available resources effectively and reduce in the total cost of the goods. Many researchers including Mishra and Bal Ram (2004), Wee et al. (2007), Kotb et al. (2011), and many others studied different variants of inventory models where quality improvement investment is the key concern. Vijayashree and Uthayakumar (2016) considered the integrated production inventory model of a vendor and buyer to examine the quality improvement, investment, and setup cost reduction so that the total profit can be maximized. Recently, Paraschos et. al. (2020) developed inventory model to suggest the policies for the optimal control, maintenance, and product quality of production by using the reinforcement learning-based approach.

Profit may turn out into loss if goods are not prevented from deterioration. According to Raafat (1991) "decay or deterioration is defined as any process that prevents an item from being used for its intended original use such as: (i) spoilage, as in perishable foodstuffs, fruits and vegetables; (ii) physical depletion, as in pilferage or evaporation of volatile liquids such as gasoline and alcohol; (iii) decay, as in radioactive substances, degradation, as in electronic components, or loss of potency as in photographic films and pharmaceutical drugs". The rate of deterioration can depend upon the nature of goods. Some goods deteriorate very fast like fruits, vegetable, milk, etc., and the rate of deterioration in some items are very low or negligible, so we can ignore the effect of it. The concept of deterioration in the inventory model has been investigated by many researchers for the last few decades (Raafat,1991; Sarkar and Sarkar, 2013). Comprehending the importance of inventory policies in practice, an integrated production-inventory-marketing model for deteriorating items was established by Goyal and Gunasekaran (1995), They considered the economic production quantity (EPQ) and economic order quantity (EOQ) for raw materials in a multi-stage production system. Andijani and Dajani (1998) considered an inventory-production system where items deteriorate at a constant rate. Inventory model with shortages was developed by Min et al. (2012); they have allowed the nature of deterioration of items as exponential. Sicilia et. al. (2014) analyzed the inventory system for deteriorating items with constant demand rate and shortages. Guchhait et al. (2015) developed a model for an inventory system by incorporating the time dependent deteriorating rates to determine the profit maximization. San-Joséa et al (2018) studied an inventory model where time-dependent demand is a function of price also. They assumed that the shortages are allowed and completely backlogged to maximize the profit per unit time. They developed an efficient algorithm to find the optimal inventory policy.

Another practical problem other than deterioration is the "limited storage facility" which has drawn the attention of some researchers. As we know that there is no space available in the main market of the city area, especially in the business hub in downtown. The location of business plays a very important role in the growth of the business as it is approachable for vendors. Retailers want some storage space in the main market area, even with more cost. The retailers are ready to pay more amount for small spaces in those markets. To avoid deterioration goods, retailers need facility of fully equipped storage area with modern machinery. There are two possible ways to get this type of storage facility, either they must own this type of facility by buying a warehouse at the market area or by taking this facility on rent (Sicilia et. al., 2014). The cost of both types of options may be different, if they are buying their own property, it will be beneficial for them as they are making their own asset, but not feasible for all. On the other hand, taking property on rent is adding extra cost on setup cost. Both the options are open for retailor and they would need to make the decision of which is beneficial to them. The objective of present study is to develop an economic production quantity (EPQ) model for deteriorating items by incorporating realistic assumptions including shortages along with fully backlogged and quality control. The feature of linear demand function is also considered.

2. MODEL DISCRIPTION

Here in this model, we develop an economic production inventory model for deteriorating items like edible items, dairy products, fruits, vegetables, grocery products, packed products, etc. In this model, we are taking different cost elements in order to determine the total cost incurred into the system. The following assumptions are made to describe the model:

Assumptions

- The demand rate is taken as a linear function of time.
- Finite production rate is considered.
- Lead time is considered to be zero.
- Model allowed shortages and fully backlogged.
- System deals with deteriorating items having rate dependent upon time.

Notations which we used in our model is as follow:

$\theta(t)$	:	Deterioration rate function.
T	:	Length of the interval after that inventories start decline.
D	:	Demand rate, D=u+vt.
I(t)	:	Inventory level at time t; t>=0.
H	:	Entire stretch of time in a system.
n	:	Number of production cycle during the entire period H.
C_1	:	Inventory shortage cost per unit time.
C_2	:	Holding cost of inventory per unit time.
C_3	:	Unit purchase cost.
C_4	:	Setup cost of the system.
C_5	:	Quality control cost.

Here, $\theta(t)= \theta t$, $C_5=Q_c t+\sum x_i k_i$; where $Q_c=q_c+q_p$, q_c=cost on quality control which includes the machine setting, labor charges, etc., q_p=cost due to pandemic situation like lockdown, x_i=per unit cost of i^{th} sample, k_i=size of i^{th} sample.

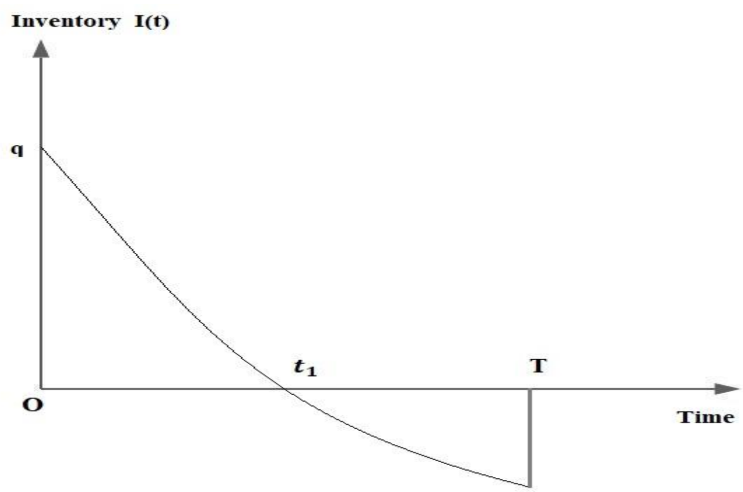

Figure 1: Change in inventory with time

The inventory model is depicted in figure 1 which demonstrates that the initially, inventory is q(t). As time increases, inventory start going down due to deterioration of items as well as demand. At time equals to t_1, the inventory level becomes zero. Then after, there is shortage of goods in the system up to time T and then inventory is instantaneously replenished at time T.

3. GOVERNING EQUATIONS AND ANALYSIS

The inventory model is formulated mathematically by framing differential equations along with boundary conditions as follows:

$$\frac{dI(t)}{dt}+\theta t I(t)=-D,\ \ 0\le t\le t_1 \tag{1}$$

$$\frac{dI(t)}{dt}=-D,\ \ t_1\le t\le T \tag{2}$$

By applying boundary condition $I(t_1)=0$ and $I(0)=q$, solution of equations (1) and (2) is given by

$$I(t)=u\left[(t_1-t)+\frac{\theta}{6}\left(t_1^3-t^3\right)-\frac{\theta}{2}t^2\left(t_1-t\right)\right]+\frac{v}{2}\left[\left(t_1^2-t^2\right)+\frac{\theta}{4}\left(t_1^4-t^4\right)-\frac{\theta}{2}t^2\left(t_1^2-t^2\right)\right];\ \ 0\le t\le t_1$$

(3)

$$I(t) = u(t_1 - t) + \frac{v}{2}\left(t_1^2 - t^2\right) \;\; ; \;\; t_1 \le t \le T \tag{4}$$

(i) Deterioration Cost:

$$\begin{aligned} DC &= C_3\left[q - \int_0^{t_1} (u + vt)dt \right] \\ &= C_3\left[\frac{u\theta t_1^3}{6} + \frac{v\theta t_1^4}{8} \right] \end{aligned} \tag{5}$$

(ii) Shortage Cost:

$$\begin{aligned} SC &= \int_{t_1}^{T} I(t)dt = -C_1 \int_{t_1}^{T} \left(u(t_1 - T) + \frac{v}{2}\left(t_1^2 - T^2\right) \right) dt \\ &= \frac{uC_1}{2}(T - t_1)^2 + \frac{vC_1}{6}\left(T^3 + 2t_1^3 - 3t_1^2 T\right) \end{aligned} \tag{6}$$

(iii) Holding Cost:

\

$$\begin{aligned} HC &= C_2 \int_0^{t_1} I(t)dt \\ &= C_2\left[u\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{12} \right) + \frac{v}{2}\left(\frac{2t_1^3}{3} + \frac{2\theta t_1^5}{15} \right) \right] \end{aligned} \tag{7}$$

(iv) Quality Control cost:

$$C_5 = Q_c t + \sum x_i k_i \tag{8}$$

Now, total cost is

$$\begin{aligned} \mathrm{TC}(\mathrm{T}, \mathrm{t}_1) &= \frac{1}{\mathrm{T}}\left(C_4 + DC + HC + SC + C_5 \right) \\ &= \frac{1}{T}\begin{pmatrix} C_4 + C_3\left[\frac{u\theta t_1^3}{6} + \frac{v\theta t_1^4}{8}\right] + \frac{uC_1}{2}(T - t_1)^2 + \frac{vC_1}{6}(T^3 + 2t_1^3 - 3t_1^2 T) \\ + C_2\left[u\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{12}\right) + \frac{v}{2}\left(\frac{2t_1^3}{3} + \frac{2\theta t_1^5}{15}\right)\right] + C_5 \end{pmatrix} \end{aligned} \tag{9}$$

In order to minimize the total cost, we take a first order partial derivative of total cost of the system with respect to time t_1 and T and equate them to zero.

$$\frac{\partial TC}{\partial t_1} = 0, \; and \; \frac{\partial TC}{\partial T} = 0$$

4. NUMERICAL ILLUSTRATIONS

In order to find the optimal minimum cost of the economic production inventory model, we have to fix parameters. The per unit costs for an item are as follows: C_1=\$30, C_2=\$10, C_3=\$15, C_4=\$600 per order, u=70, v=20, θ=0.1, Q_c=\$10, x_1=\$5, x_2=\$6, k_1=30, k_2=30. The time horizon is measured in months. In order to analyze the effect of deterioration rate, unit purchase cost, quality control cost, shortage cost on total cost, we obtain numerical results. The convex nature of the total cost function is shown in the fig. 2. This ensures that there exists the time t_1 and T for which the cost function will be minimum.

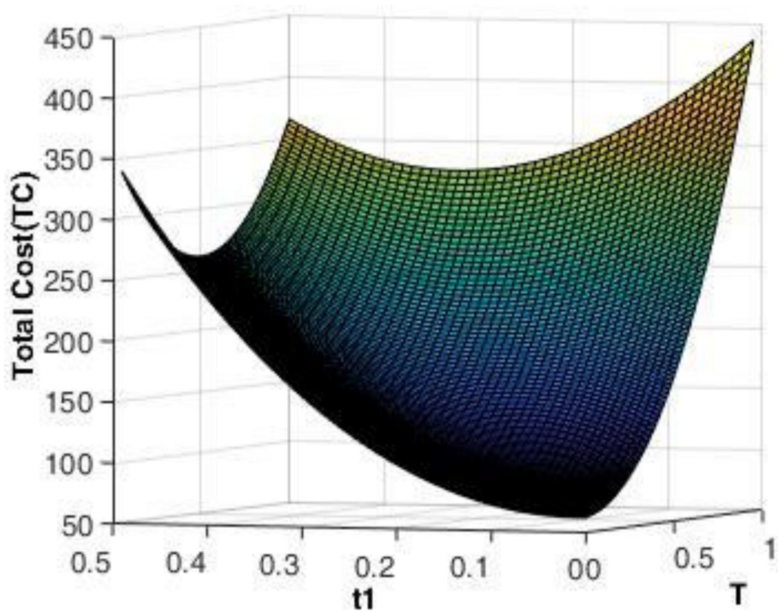

Figure 2: Convexity of the total cost function

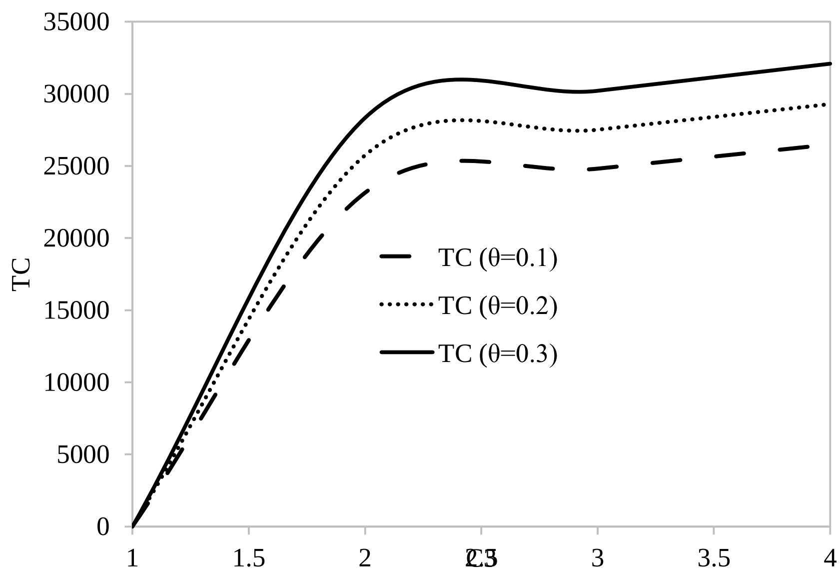

Figure 3: Total cost vs. unit purchase cost with different deterioration rate

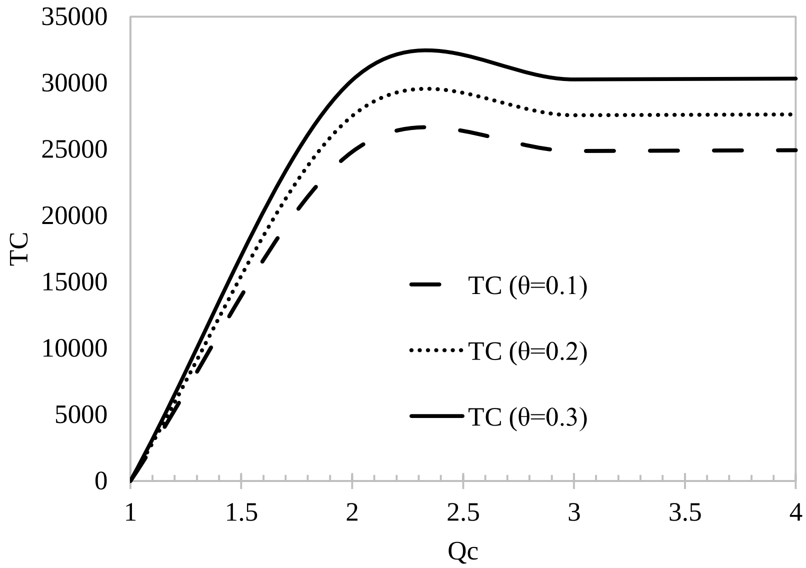

Figure 4: Total cost vs. quality control cost with different deterioration rate

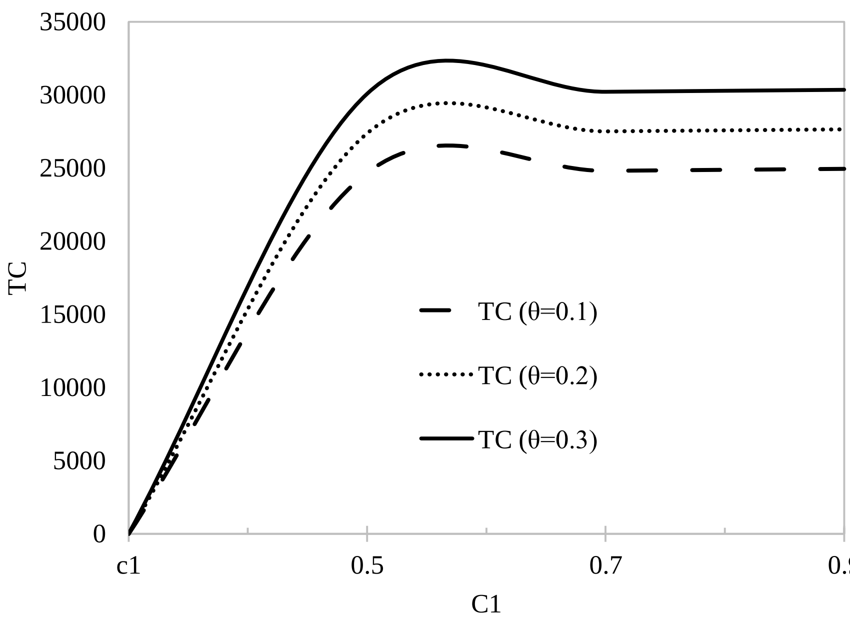

Figure 5: Total cost vs. inventory shortage cost with different deterioration rate

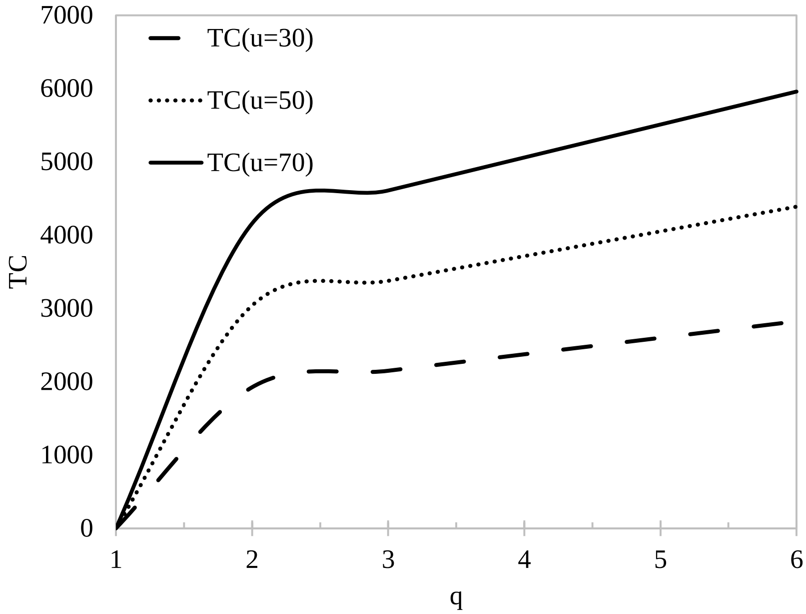

Figure 6: Total cost vs. deterioration rate with different parameter u

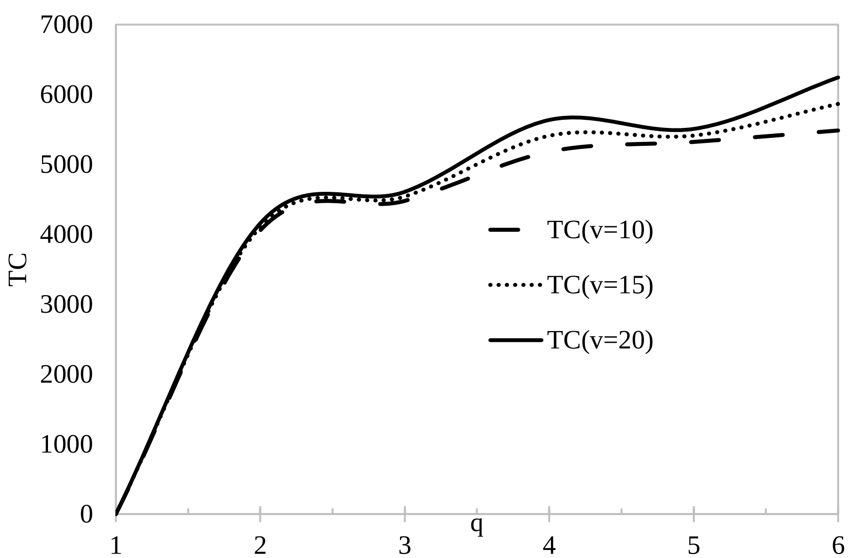

Figure 7: Total cost vs. deterioration rate with different parameter v

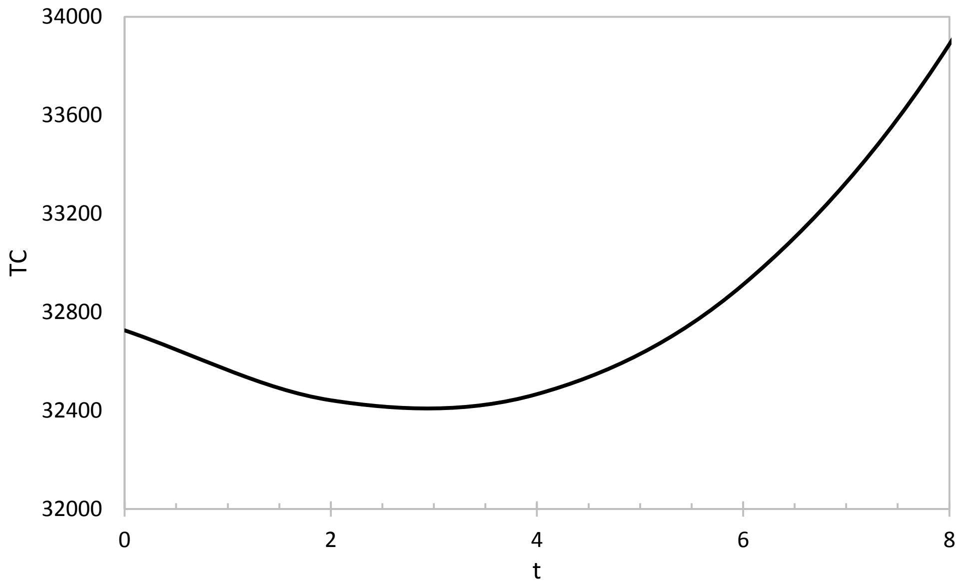

Figure 8: Total cost vs. time

By using the quasi-newton method to optimizing the cost function, the optimal cycle time is obtained as: $t_1 = 1.06 \text{ months}, \mathrm{T} = 1.46 \text{ months}$. The total optimal cost is $TC^* = \$1148.78$.

Figure 3 shows the relation between the unit purchase cost and total cost with different deterioration rate. We observe the increasing trend for both the parameters. From figure 4 we see the effect of quality control cost with different deterioration rate and total cost. The total cost is increased with the increment in the quality control cost. We see that the total cost of the system increases by increasing the cost of quality control. It can be concluded that this is the necessary factor by which any businessman ensures the quality of products and run their business for long time and in the time of pandemic.

The effect of inventory shortage cost with different deterioration rate on total cost is depicted in figure 5. As deterioration rate and shortage cost increase, we see that the total cost also increases. Thus, to manage the total cost of products, the manufacturer has to work line to reduce the deterioration rate. Figure 6 exhibits the effect of deteriorating rate for different rate of parameter ‘u’ on the total cost. From this figure, we observe that as deterioration rate increases, there is increase in total cost. The effect of deterioration rate on total cost with different rate of parameter ‘v’ is shown in figure 7. As we increase the value of v, the total cost also goes up. The effect of time-period on total cost is also a significant factor in inventory system; this effect is depicted in figure 8. If we increase the time-period, the total cost of the inventory system also ramps up.

5. CONCLUSION

In this paper, we have studied the inventory model for deteriorating items to analyze the economic production along with quality control. The quality control cost is considered as the sum of the cost incurred in quality inspection and additional cost due to pandemic scenario of covid-19, which also enhances the goodwill of the company. By including the concept of deteriorating products, which deteriorate with time and also affect the inventory as time grows up, our model is helpful for any company to maintain their quality to run their business smoothly. Further, our model can be extended in future by using trade credit in the system and/or by taking production function which includes quality control cost.

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