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COST INSPECTION OF UNRELIABLE M^X/G/1 QUEUE WITH FIXED ROUNDS OF BERNOULLI FEEDBACK

Shweta Upadhyaya, Amity University, Noida, India (supadhyay@amity.edu)

ABSTRACT

An M^X/G/1systemis analyzed whereintheservice provider or server which is unreliable can provide service in two phases wherein server makesfinite rounds of Bernoulli feedbacks. Here, customers arrive with their arrival rates dependent upon server status. Our interest for selecting this model arises from our day-to-dayreal-life situations encountered in telecommunication networks, service systems, computer systems etc. where server may face breakdown at any stage of service.The prominent performance indices like probability generating function of the system sizeand other in steady state are calculated. The ideal values of fewvariables such that the system cost is minimized are also obtained. Finally, we provide numerical examples to simulate the model with the real time systems.

Keywords: Two phase service, Bernoulli feedback, Batch arrival, Server failure, Cost Optimization.

1. INTRODUCTION

There are many queueing congestion situations in which customers make repeated attempts to complete its service This can be visualized in many practical situations in packet switching networks, CCN's (computer and communication networks) and data communication systems. For example, flow of messages in data transmission, flow of information (or packets) between various sources and destinations in Internet systems, flow of calls in a telephone call/contact centers or requests for money transfer in ATM machines. This happens because of blocking of these systems under limited resources or server capacity. In the previous literature, we find the concept of "instantaneous feedback" where customer has to go to the initial point and wait there till the service provider is unavailable to render the next service (cf. Disney et al., 1980) but in this study, instead of taking single feedback, an arriving customer may undergo finite rounds ofimmediate Bernoulli feedbacks if they are not satisfied with its service.

Most of the times, it happens that the service provider may fail to serve the arriving customers. Once server fails, it can be repaired by the repairmen available in the system. This situation is known as 'unreliable server' or 'server breakdown' in queueing context. For example, if there is some problem in the computer system such as hardware failure or software failure, it may be repaired by the repairmen available in the system. Upadhyaya (2014) gave performance indices for cluster arrival recurrent queue with Bernoulli feedback. Praveen and Begum (2014) have investigated bulk arrival model including retrials under Bernoulli vacationalong with multi-optional services. Singh et al. (2016) have scrutinized batch arrival unreliable recurrent model which has an option of additional service. Li et al. (2017) considered a negative arrival along with normal arriving customers in an M/G/1 retrial queue. They have done the optimization of the system as well. Jain and Meena (2018) have studied a Markov model for machine repair problem consisting of failed heterogeneous servers. Ammar et al. (2019) has investigated a M/G/1 retrial queue with priority customers along with disasters and working breakdown services. Upadhyaya (2020) have derived fruitful performance indices for batch arrival priority system with negative arrivals and breakdown.Yen et al. (2020) has analyzed M/G/1 queues with N policy and working failure. Rajadurai et al. (2020) analyzed an M^X/G/1 queue with retrials, preemptive priority and feedback along with balking of the customers due to disasters and working breakdowns.

In this paper, we investigate queueing characteristics of a two-phase unreliable M^X/G/1 queue with countable rounds of Bernoulli feedback under arrival rates that are state dependent. The remaining paper is organized as follows.Insection 2, we describe the model by giving suitable assumptions andsymbols. Section 3 is devoted to obtain system size distribution of the developed model. The followingresults are then used in section 4, to obtain various useful performance measures for the system.Section 5 presented cost analysis of the developed model and it also includes some numericalresults. At last, we give remarks for conclusion and scope for future refence of the work in section 6.

2. MODEL DESCRIPTION

All customers arrive following the Poisson process with rate depending on the state of service provider as

$$\lambda_n = \begin{cases} \lambda_0, & \text{for server being idle} \\ \lambda_1, & \text{for server being busy or in repair during FPS(first phase service)} \\ \lambda_2, & \text{for server being busy or in repair during SPS(second phase service)} \end{cases}$$

We denotebatch size by random variable X, with probability function $\Pr\{X = m\} = c_m; \text{m} \geq 1$ and probability generating function C(z)= $\sum_{m=1}^{\infty} c_m z^m$ with $|z| < 1$.We denote first and second moments of X by c_1 and c_2, respectively, so that c_1=E(X) and c_2= $E(X^2)$.

In the present investigation, serviceis provided in two stages denoted by FES (first essential service)and SPS (second phase service). Each unit in the system first optsfor FES and then SPSwhich is optional dependingupon thechoice of an arriving customer. After the accomplishment of the first stage,the customer can go for any of***J***($1 \leq J \leq K$) optional services with probability p_J, otherwise leaves the system with complementary probability $1 - p_J$. The complete service (first phase and/or second phase) is provided by the same server. We assumed the customers that have completed their first stage service to either feedback instantly with probability α_1 or exit straight away with complementary probability $1 - \alpha_1$.The units may enter the queue again for the next r feedback phase by coming into the essential phase of service with probability α_2 ($\alpha_2 < \alpha_1$); after finishing their feedback service, they leave the system with complimentary probability $1 - \alpha_2$.This process is repeated again and again till an arriving customer completes ***m*** rounds of feedback. The next arriving unit in the queue can enter the system only if the previous customer finishes his/her satisfied number of feedbacks. There are***K*** numbers of optional servicesavailable in the queueing system.Themodel followsFCFS discipline.The server may fail eitherduring FPS or SPS and is sent immediately for the purpose ofrepairby the repairmen available in the system.When server breakdown occurs, the customer in the service holds back for the service provider which is repaired to finish off the left over service.Service times and repair times are general distributed. We define other notations of the model as follows:

2.1 Notations

$B(x)$	:	Distribution function of service time during essential phase of service.
$B^*(S)$	:	Laplace Steiljes transform for the distribution function of service time during essential phase service.
$E[B_0] = -B^{*'}(0)$	:	First moment of service time distribution during essential phase service.
$E[B_0^2] = B^{*''}(0)$	:	Second moment of service timedistribution during essential phase of service.
$B_J(x)$, (1≤J≤K)	:	Distribution functions of the service time during Jth optional phase of service.
$B_J^*(S)$, (1≤J≤K)	:	Laplace Steiljes transforms forthe distribution function of the service time during Jth optional phase of service.
$\mu_J(x)$, (1≤J≤K)	:	Hazard rate function for service time during Jth optional phase of service.
$E[B_J] = -B_J^{*'}(0)$	:	First moment of service time distribution during essential phase of service.

(1≤J≤K)		
$E\left[B_J^2\right] = B_J^{*''}(0)$ (1≤J≤K)	:	Second moment of service time distribution during J[th] optional phase of service
$D(x)$	:	Distributionfunction of time for repair of the server when it's on breakdowns while on essential phase of service.
$D^*(S)$	:	Laplace Steiljes transform for the distribution function of time for repair of the server when it breakdowns while on essential phase of service
$E\left[D_0\right] = -D^{*'}(0)$	:	First moment of time for repair distributionwhen server breakdowns while on essential phase of service.
$E\left[D_0{}^2\right] = D^{*''}(0)$	:	Second moment of time for repair distribution when server breakdowns while on essential phase of service.
$E\left[D_J\right] = -D_J^{*'}(0)$ (1≤J≤K)	:	First moment of time for repair distribution when server breakdowns while on J[th] optional phase of service.
$E\left[D_J^2\right] = D_J^{*''}(0)$ (1≤J≤K)	:	Second moment of repair time distribution when server breakdowns J[th] during optional phase of service.

3. SYSTEM SIZE DISTRIBUTION

Let P (t) showsthe server's state at time t, then

$$P(t) = \begin{cases} 0, & \text{when server being idle.} \\ 1, & \text{when server is providing essential stage service.} \\ 2, & \text{when server is providing optional stage service.} \\ 3, & \text{if the server is in repair during essential phase of service.} \\ 4, & \text{if the server is in repair under optional phase of service.} \end{cases}$$

Let Q (t) be the customers' number in the system at time t, K(t) denotes the elapsed service time of a customer presently receiving service during FES at time t, $L_J(t)$ denotes the elapsed service time of the unit presently receiving J[th]optional service during second stage of service at time t, M (t) denotes the elapsed repair time of a server when breakdown during FES and $N_J(t)$ denotes the elapsed repair time of a of a server when breakdown during SPS.

Define
$$\Psi(t) = \begin{cases} 0, \mathrm{P}(t) = 0 \\ \mathrm{K}(t), \mathrm{P}(t) = 1 \\ L_J(t), \mathrm{P}(t) = 2 \\ M(t), \mathrm{P}(t) = 3 \\ N_J(t), \mathrm{P}(t) = 4 \end{cases}$$

Then {P (t), Q (t), $\Psi(t)$, t≥0} be a continuous time Markov process. We construct the following probability functions as follows:

$U_{0,0}$ = Prob {P (t) = 0, Q (t) = 0}

$V_{i,n}(x,t)dx$ = Prob {P(t) = 1, Q (t) = n, $x \le K(t) \le x + dx$ }; $0 \le \mathrm{i} \le \mathrm{m}-1$.

$U_{i,n}^J(x,t)dx$ = Prob {P (t) =2, Q (t) =n, $x \le L_J(t) \le x + dx$ }; $1 \le \mathrm{J} \le \mathrm{K}, 0 \le \mathrm{i} \le \mathrm{m}-1$.

$W_{i,n}^0(x,y,t)dx$ = Prob {P (t) = 3, Q (t) =n, $K(t) = x,\ y \le M(t) \le y + dy$ }; $1 \le \mathrm{J} \le \mathrm{K}, 0 \le \mathrm{i} \le \mathrm{m}-1$.

$W_{i,n}^J(x,y,t)dx$ =Prob {P (t) =4, Q (t) =n, $L_J(t) = x,\ y \le N_J(t) \le y + dy$ };$1 \le \mathrm{J} \le \mathrm{K}, 0 \le \mathrm{i} \le \mathrm{m}-1$.

3.1 Steady State Equations

$$\lambda_0 U_{0,0} = \sum_{S=0}^{m-2}(1-\alpha_{S+1})\int_0^\infty \sum_{J=1}^{K} U_{S,1}^{J}(x)\mu_J(x)dx + \sum_{j=1}^{k}\int_0^\infty U_{m-1,1}^{J}(x)\mu_J(x)dx \quad (1)$$

$$\frac{d}{dx}V_{i,n}(x) = -\{\lambda_1 + \mu(x) + b_0\}V_{i,n}(x) + (1-\delta_{1,n})\lambda_1 \sum_{x=1}^{n} c_x V_{i,n-x}(x) + \int_0^\infty \beta_0(y)W_{i,n}^{0}(x,y)dy \quad ; 0\leq i \leq m-1 \quad (2)$$

$$\frac{d}{dx}U_{i,n}^{J}(x) = -\{\lambda_2 + \mu_J(x) + b_J\}U_{i,n}^{J}(x) + (1-\delta_{1,n})\lambda_2 \sum_{x=1}^{\infty} c_x U_{i,n-1}^{J}(x) + \int_0^\infty \beta_J(y)W_{i,n}^{J}(x,y)dy$$
$$;1 \leq J \leq K, 0 \leq i \leq m-1 \quad (3)$$

$$\frac{\partial}{\partial x}W_{i,n}^{0}(x,y) = -\{\lambda_1 + \beta(y)W_{i,n}^{0}(x,y) + \lambda_1 \sum_{x=1}^{n} c_x W_{i,n-x}^{0}(x,y) \quad ; 0\leq i\leq m-1,\ n\geq 1 \quad (4)$$

$$\frac{\partial}{\partial x}W_{i,n}^{J}(x,y) = -\{\lambda_2 + \beta_J(y)W_{i,n}^{J}(x,y) + \lambda_2 \sum_{x=1}^{n} c_x W_{i,n-x}^{J}(x,y) \quad ;1 \leq J \leq K, 0 \leq i \leq m-1 \quad (5)$$

Now we define boundary conditions as follows:

$$V_{o,1}(0) = \lambda_0 U_{0,0} + \sum_{S=0}^{m-2}(1-\alpha_{S+1})\int_0^\infty \sum_{J=1}^{K} U_{S,2}^{J}(x)\mu_J(x)dx + \sum_{j=1}^{k}\int_0^\infty U_{m-1,2}^{J}(x)\mu_J(x)dx \quad (6)$$

$$V_{0,n}(0) = \sum_{s=0}^{m-2}(1-\alpha_{s+1})\int_0^\infty \sum_{J=1}^{K} U_{s,n+1}^{J}(x)\mu_J(x)dx + \sum_{J=1}^{K}\int_0^\infty U_{m-1,n+1}^{J}(x)\mu_J(x)dx \ ; n\geq 2 \quad (7)$$

$$V_{i,n}(0) = q_i \int_0^\infty \sum_{J=1}^{K} U_{i-1,n}^{J}(x)\mu_J(x)dx; 1 \leq i \leq m-1, n \geq 1 \quad (8)$$

$$U_{i,n}^{J}(0) = p_J \int_0^\infty V_{i,n}(x)\mu(x)dx; 1 \leq J \leq K, 0 \leq i \leq m-1, n \geq 1 \quad (9)$$

$$W_{i,n}^{0}(0) = b_0 V_{i,n}(x); n \geq 1, 0 \leq i \leq m-1 \quad (10)$$

$$W_{i,n}^{J}(0) = b_J U_{i,n}^{J}(x); n \geq 1;\ 1 \leq J \leq K, 0 \leq i \leq m-1 \quad (11)$$

The normalizing condition is given by

$$U_{0,0} + V_0(1) + \sum_{i=1}^{m-1} V_i(1) + \sum_{J=1}^{K} U_0^{J}(1) + \sum_{i=1}^{m-1}\sum_{J=1}^{K} U_i^{J}(1) + W_0^{(0)}(1) + \sum_{i=1}^{m-1} W_i^{(0)}(1) + \sum_{J=1}^{K} W_0^{J}(1) + \sum_{i=1}^{m-1}\sum_{J=1}^{K} W_i^{J}(1) = 1 \quad (12)$$

To obtain the performance indices, via probability generating function (PGF) technique, the partial probability generating functionsfor distinct modes of the server are defined as follows:

For $1 \leq i \leq m-1, 1 \leq J \leq K$

$$V_i(x,z) = \sum_{n=1}^{\infty} V_{i,n}(x)z^n \ ; U_i^{J}(x,z) = \sum_{n=1}^{\infty} U_{i,n}^{J}(x,z)z^n \ ; W_i^{0}(x,z) = \sum_{n=1}^{\infty} W_{i,n}^{0}(x,z)z^n$$

$$W_i^{J}(x,z) = \sum_{n=1}^{\infty} W_{i,n}^{J}(x,z)z^n \ ; C(z) = \sum_{n=1}^{\infty} c_n z^n$$

Lemma 1: The marginal pgf for distinct states of the system are given by

$$V_0(z)=\int_0^\infty V_0(x,z)dx=\frac{\lambda_0 z(z-1)(1-B^*((h_1(z))U_{0,0}}{h_1(z)(z-\varphi(z))}$$

$$V_i(z)=\int_0^\infty V_i(x,z)dx=\frac{\lambda_0 z(z-1)(1-B^*((h_1(z))\prod_{J=1}^{i} q_J\left(E(z)\right)^i U_{0,0}}{h_1(z)(z-\varphi(z))} \quad ;1\le \mathrm{i}\le \mathrm{m}-1$$

$$U_0^J(z)=\int_0^\infty U_0^J(x,z)dx=\frac{\mathrm{p}_J\lambda_0 \mathrm{z}(\mathrm{z}-1)\mathrm{B}^*(\mathrm{h}_1(\mathrm{z}))(1-\mathrm{B}_J^*(\mathrm{h}_2(\mathrm{z}))\mathrm{U}_{0,0}}{(\mathrm{z}-\varphi(\mathrm{z}))\mathrm{h}_2(\mathrm{z})} \quad ;\ 1\le \mathrm{J}\le \mathrm{K}$$

$$U_i^J(z)=\int_0^\infty U_i^J(x,z)dx=\frac{p_J\{\prod_{J=1}^{i} q_J\}\lambda_0 z(z-1)B^*(h_1(z))(1-B_J^*(h_2(z)E^i(z)U_{0,0}}{h_2(z)(z-\varphi(z))} \quad ;\ 0\le \mathrm{i}\le \mathrm{m}-1,\ 1\le \mathrm{J}\le \mathrm{K}$$

$$W_0^0(z)=\int_0^\infty W_0^0(x,z)dx=\frac{b_0\lambda_0 z(z-1)(1-B^*(h_1(z))(1-D_0^*(\lambda_1(1-C(z)))U_{0,0}}{h_1(z)(\lambda_1(1-C(z))(z-\varphi(z)}$$

$$W_i^0(z)=\int_0^\infty W_i^0(x,z)dx$$

$$=\frac{b_0\lambda_0 z(z-1)(\prod_{J=1}^{i} q_J)(1-B^*(h_1(z))E(z)^J(1-D_0^*(\lambda_1(1-C(z)))U_{0,0}}{h_1(z)\lambda_1(1-C(z))(z-\varphi(z)} \quad ;\ 0\le \mathrm{i}\le \mathrm{m}-1$$

$$W_0^J(z)=\int_0^\infty W_0^J(x,z)dx$$

$$=\frac{b_J p_J\lambda_0 z(z-1)B^*(h_1(z))(1-B_J^*(h_2(z))(1-D_J^*(\lambda_2(1-C(z))U_{0,0}}{h_2(z)\lambda_2(1-C(z))(z-\varphi(z)} \quad ;\ 1\le \mathrm{J}\le K$$

$$W_i^J(z)=\int_0^\infty W_i^J(x,z)dx$$

$$=\frac{b_J p_J\lambda_0 z(z-1)(\prod_{J=1}^{i} q_J)B^*(h_1(z)))(1-B_J^*(h_2(z))(1-D_J^*(\lambda_2 1-C(z)))U_{0,0}}{h_2(z)\lambda_2(1-C(z))(z-\varphi(z)} \quad ;\ 0\le \mathrm{i}\le m-1, 1\le \mathrm{J}\le K$$

where $h_1(z)=\lambda_1(1-C(z))+b_0(1-B_0^*(\lambda_1(1-C(z))$ and $h_2(z)=\lambda_2(1-C(z))+b_J(1-B_J^*(\lambda_1(1-C(z))$

$$\varphi(z)=\{(1-\alpha_1)E(z)+\sum_{s=1}^{m-1}(1-q_{s+1})q_1q_2\_\_\_\_\_\_\_\_ q_s\left(E(z)\right)^{s+1}\}\ \text{with}\ q_m=0$$

$$E(z)=\sum_{J=1}^{K} p_J B^*(h_1(z))B_i^*(h_2(z)).$$

Proof: Multiplying equations (2)-(11) withappropriate powers of z and then taking summation over ‘n’ and thereafter performing some algebraic computations we obtain the above results.

Theorem 1: The PGF of average system size is

$$U(z)=U_{0,0}+V_0(z)+\sum_{i=1}^{m-1}V_i(z)+\sum_{J=1}^{K}U_0^J(z)+\sum_{i=1}^{m-1}\sum_{J=1}^{K}U_i^J(z)+W_0^{(0)}(z)+\sum_{i=1}^{m-1}W_i^{(0)}(z)+\sum_{J=1}^{K}W_0^J(z)+\sum_{i=1}^{m-1}\sum_{J=1}^{K}W_i^J(z)$$

Proof: Summing up all the marginal PGF of distinct states of the server, we obtain the above result.

4. PERFORMANCE MEASURES

The steady state probabilities for different states of the model are

- Probability that service provider is idle is given by

$$U_{0,0}=\frac{1-\varphi^{'}(1)}{(1-\varphi^{'}(1)+(\tau)\{1+\sum_{i=1}^{m-1}\prod_{J=1}^{i}q_J\}}$$

$$\tau=\lambda_0\left[\{1+b_0E(D_0)\}E(B)+\sum_{J=1}^{K}p_J\{1+b_JE(D_J)\}E(B_J)\right];$$

$$\varphi^{'}(1)=E^{'}(1)\left[1+q_1+q_1q_2+\ldots\ldots\ldots\ldots\ldots\ldots+q_1q_2\ldots\ldots q_{m-1}\right]$$

$$E^{'}(1)=c_1\left[\lambda_1\{1+b_0E(D_0)E(B)+\lambda_2\sum_{J=1}^{K}p_J\{1+b_JE(D_J)\}E(B_J)\right].$$

- Probability that service provider is busy with FES under 0th feedback is obtained using

$$P[F_0]=\frac{\lambda_0E(B_0)U_{0,0}}{1-\varphi^{'}(1)}$$

- Probability that the service provider is busy with FES under ith feedback is given by

$$P[F_i]=\sum_{i=1}^{m-1}\frac{\lambda_0\{\prod_{J=1}^{i}q_J\}E(B_0)U_{0,0}}{1-\varphi^{'}(1)}\ ;1\le i\le m-1$$

- Probability that service provider is busy with 0th feedback of SPS can be found using

$$P[S_0]=\sum_{J=1}^{K}\frac{\lambda_0p_JE(B_J)U_{0,0}}{1-\varphi^{'}(1)}$$

- Probability that service provider is busy withith feedback of SPS is given by

$$P[S_i]=\frac{\lambda_0\{\prod_{J=1}^{i}q_J\}p_JE(B_J)U_{0,0}}{1-\varphi^{'}(1)}\ ;1\le i\le m-1$$

- Probability that the service provider is under repair when breakdown during 0th feedback of FES is

$$P\left[R_0^{(0)}\right]\frac{\lambda_0\alpha_0 E(B)E(D_0)U_{0,0}}{1-\varphi'(1)}$$

- Probability that theservice provider is under repair when breakdown during i^th feedback of FES is computed as

$$P\left[R_i^{(0)}\right]=\sum_{i=1}^{m-1}\frac{\lambda_0\alpha_0\{\prod_{J=1}^{i}q_J\}E(B)E(D_0)U_{0,0}}{1-\varphi'(1)}\ ;1\le \text{i}\le m-1$$

- Probability that the service provider is under repair when breakdown during 0^th feedback of SPSis found as

$$P\left[R_0^{(J)}\right]=\sum_{J=0}^{K}\frac{\lambda_0\alpha_J p_J E(B_J)E(D_J)U_{0,0}}{1-\varphi'(1)}$$

- Probability that the service provider is under repair when breakdown during i^th feedback of SPS is given by

$$P\left[R_i^{(J)}\right]=\sum_{i=0}^{m-1}\sum_{J=1}^{K}\frac{\lambda_0\alpha_J p_J\{\prod_{J=1}^{i}q_J\}E(B_J)E(D_J)U_{0,0}}{1-\varphi'(1)}\ ;1\le \text{i}\le m-1$$

- The time spent by the service provider in complete service during 0^th feedback is given by
  P (FR_0) = $E'(1)$

- The time spent by the service providerin complete service during i^th feedback is given by

$$\text{P (FR}_i\text{)} = \left(\prod_{J=1}^{i}q_J\right)E'(1)\ ;1\le \text{i}\le m-1$$

Theorem 2: The average number of customers in the system is given by

$$U'(1)=U_{0,0}+V_0'(1)+\sum_{i=1}^{m-1}V_i'(1)+\sum_{J=1}^{K}U_0^{(J)'}(1)+\sum_{i=1}^{m-1}\sum_{J=1}^{K}U_i^{(J)'}(1)+W_0^{(0)'}(1)+\sum_{i=1}^{m-1}W_i^{(0)'}(1)+\sum_{J=1}^{K}W_0^{(J)'}(1)+\sum_{i=1}^{m-1}\sum_{J=1}^{K}W_i^{(J)'}(1)$$

where, $V_0'(1)=\frac{\lambda_0 U_{0,0}(D''N'''-N''D''')}{3(D'')^2}$; $N''=2E(B)h_1'(1)$; $N'''=3h_1'(1)\left[2E(B)-h_1'(1)E(B^2)\right]$

$$D''=2h_1'(1)(1-\varphi'(1))\ ;D'''=3\left[(1-\varphi'(1)h_1''(1)-\varphi'(1)h_1'(1)\right]$$

$$V_i'(1)=\frac{\lambda_0 U_{0,0}\prod_{J=1}^{i}q_J(D_1''N_1'''-N_1''D_1''')}{3(D_1'')^2}\quad ;\ 0\le \text{i}\le \text{m-1}\ ;N_1''=2h_1'(1)E(B)$$

$$N_1'''=3\left[2(1+J)E(B)-h_1'(1)E(B^2)\right]h_1'(1)\ ;D_1''=2h_1'(1)(1-\varphi'(1))$$

$$D_1'''=3\left[(1-\varphi'(1)h_1''(1)-\varphi''(1)h_1'(1)\right]$$

$$U_0^{(J)'}(1)=\frac{\lambda_0 p_J U_{0,0}(D_2''N_2'''-N_2''D_2''')}{3(D_2'')^2}\quad ;\ 1\le \text{J}\le \text{K}$$

$$N_2''=2h_2'(1)E(B_J)\ ;N_2'''=3\left[2h_2'(1)E(B_J)\{1-E(B)h_1'(1)\}+h_2''(1)E(B_J)-(h_2'(1))^2E(B_J^2)\right]$$

$$D_2''=2h_2'(1)(1-\varphi'(1))\ ;D_2'''=3\left[1-\varphi'(1)h_2''(1)-\varphi''(1)h_2'(1)\right]$$

$$U_i^{(J)'}(1) = \frac{\lambda_0 p_J U_{0,0} \prod_{J=1}^{i} q_J (D_3^{''} N_3^{'''} - N_3^{''} D_3^{'''})}{3(D_3^{''})^2} \;;\; 0 \le i \le m-1, 1 \le J \le K$$

$$N_3^{''} = 2h_2^{'}(1)E(B_J) \;;\; N_3^{'''} = 3\left[2h_2^{'}(1)E(B_J)\{J+1-E(B)h_1^{'}(1)\} + h_2^{''}(1)E(B_J) - (h_2^{'}(1))^2 E(B_J^2)\right]$$

$$D_3^{''} = 2h_2^{'}(1)(1-\varphi^{'}(1)) \;\; D_3^{'''} = 3\left[1-\varphi^{'}(1)h_2^{''}(1) - \varphi^{''}(1)h_2^{'}(1)\right]$$

$$D_4^{'''} = -6c_1h_1^{'}(1)(1-\varphi^{'}(1)) \quad ; N_4^{'''} = -6\lambda_1 c_1 E(B)E(D_0)h_1^{'}(1)$$

$$W_0^{(0)'}(1) = \frac{\lambda_0 b_0 U_{0,0}(D_4^{'''} N_4^{1V} - N_4^{'''} D_4^{1V})}{4(D_4^{'''})^2}$$

$$N_4^{1V} = 12\left[-\lambda_1\{c_2h_1^{'}(1) + c_1h_1^{''}(1) + 2c_1h_1^{'}(1)\}E(B)E(D_0) - \lambda_1^2c_1^2h_1^{'}(1)E(B)E(D^2)\right]$$

$$+\lambda_1 C_1(h_1^{'}(1))^2 E(B^2)E(D_0)$$

$$D_4^{1V} = 12\left[-c_1\{h_1^{''}(1)(1-\varphi^{'}(1)) - h_1^{'}(1)\varphi^{''}(1)\} - c_2h_1^{'}(1)(1-\varphi^{'}(1))\right]$$

$$W_i^{(0)'}(1) = \frac{\lambda_0 b_0 U_{0,0} \prod_{J=1}^{i} q_J (D_5^{'''} N_5^{1V} - N_5^{'''} D_5^{1V})}{4\lambda_1 (D_5^{'''})^2} \;;\; 0 \le \mathrm{i} \le m-1$$

$$N_5^{'''} = -6\lambda_1 c_1 E(B)E(D_0)h_1^{'}(1)$$

$$N_5^{1V} = -\lambda_1 c_1 24(J+1)E(B_0)E(D_0) + 12\left[-\lambda_1\{c_2h_1^{'}(1) + c_1h_1^{''}(1) + 2c_1h_1^{'}(1)\}E(B)E(D_0) - \lambda_1^2c_1^2h_1^{'}(1)E(B)E(D^2)\right]$$

$$+\lambda_1 C_1(h_1^{'}(1))^2 E(B^2)E(D_0)$$

$$D_5^{'''} = -6c_1h_1^{'}(1)(1-\varphi^{'}(1)) \;\; D_5^{1V} = 12\left[-c_1\{h_1^{''}(1)(1-\varphi^{'}(1)) - h_1^{'}(1)\varphi^{''}(1)\} - c_2h_1^{'}(1)(1-\varphi^{'}(1))\right]$$

$$W_0^{(J)'}(1) = \frac{\lambda_0 b_J p_J U_{0,0}(D_6^{'''} N_6^{1V} - N_6^{'''} D_6^{1V})}{\lambda_2 4(D_6^{'''})^2} \;;\; 1 \le \mathrm{J} \le K$$

$$D_6^{'''} = -6c_1h_2^{'}(1)(1-\varphi^{'}(1)) \;;\; N_6^{'''} = -6c_1\lambda_2 h_2^{'}(1)E(B_J)E(D_J)$$

$$N_6^{1V} = 8(3\lambda_2c_1h_1^{'}(1)h_2^{'}(1)E(B)E(B_J)E(R_J) - \lambda_2c_1\{h_2^{''}(1) + 2h_2^{'}(1)\}E(B_J)E(D_J) + \lambda_2c_1(h_2^{'}(1))^2E(B_J^2)E(D_J))$$

$$D_6^{1V} = 12\left[-c_1\{h_2^{''}(1)(1-\varphi^{'}(1)) - h_2^{'}(1)\varphi^{''}(1)\} - c_2h_1^{'}(1)(1-\varphi^{'}(1))\right]$$

$$W_i^{(J)'}(1) = \frac{\lambda_0 b_J p_J U_{0,0}\left(\prod_{J=1}^{i} q_J\right)(D_7^{'''} N_7^{1V} - N_7^{'''} D_7^{1V})}{\lambda_2 4(D_7^{'''})^2} \;;\; 0 \le i \le m-1, \; 1 \le \mathrm{J} \le \mathrm{K}$$

$$D_7^{'''} = -6c_1h_2^{'}(1)(1-\varphi^{'}(1)) \;;\; N_7^{'''} = -6c_1\lambda_2h_2^{'}(1)E(B_J)E(D_J)$$

$$N_7^{1V} = \{-24\lambda_2c_1(J+1)_1h_2^{'}(1) - 12\lambda_2c_1h_2^{''}(1) - 12\lambda_2c_2h_2^{'}(1)\}E(B_J)E(D_J) + 24\lambda_2c_1h_1^{'}(1)h_2^{'}(1)E(B)E(B)_J E(D_J)$$

$$+12\lambda_2c_1(h_1^{'}(1))^2E(B_J^2)E(D_J) - 12\lambda_2^2c_1^2h_2^{'}(1)E(B_J)E(D_J^2)$$

$$D_7^{1V} = 12\left[-c_1\{h_2^{''}(1)(1-\varphi^{'}(1)) - h_2^{'}(1)\varphi^{''}(1)\} - c_2h_1^{'}(1)(1-\varphi^{'}(1))\right]$$

Proof: Differentiating all marginal generating functions with respect to z and then taking lim z tends to 1, and after that summing all these, we obtain the above result.

5. COST OPTIMIZATION AND SENSITIVITY ANALYSIS

To make the system cost effective, we have designed a cost model for an $M^X/G/1$ queue wherein server undergoesfinite rounds of Bernoulli feedbacks and arrival rates are assumed to be state dependent. This may be used in handling issues related to finance, of many industrial organizations and service systems under technology oriented economic constraints. Our main aim is to decide the joint ideal estimations of most delicate parameters p_1 and μ_1 i.e. ***(p_1^*, μ_1^*)*** simultaneously to reduce the total system cost. Here, $\mu_1 = E(B_0)$ and $\gamma_1 = E(D_0)$.

Table 1: Cost sets for optimal policy				
Cost sets	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$
Set 1	\$5	\$120	\$8	\$150
Set 2	\$5	\$120	\$9	\$150
Set 3	\$5	\$120	\$10	\$145

All the cost elements related with the cost function are as follows:

$\tilde{C}_1$: Cost of holding per unit time for everyunitin the system

$\tilde{C}_2$: Cost per unit time while the server beingactive and is operating

$\tilde{C}_3$: Setup cost per busy cycle

$\tilde{C}_4$: Startup cost per unit time before initiating the service for doing the preliminary work.

Table 2: The optimum values (p_1^*, μ_1^*) and their minimum cost E[TC*]				
Cost Sets	**(b_1, γ_1)=(0.04,0.2)**		**(b_1, γ_1)=(0.05,0.25)**	
	(p_1^*, μ_1^*)	**E[TC*]**	**(p_1^*, μ_1^*)**	**E[TC*]**
Set 1	(0.14, 2)	\$139.61	(0.14, 2)	\$139.13
Set 2	(0.14, 2)	\$139.77	(0.14, 2)	\$139.29
Set 3	(0.14, 2)	\$139.09	(0.14, 2)	\$139.62

We try to construct the function for the total cost expected per unit time as follows:

$$E[TC(p_1, \mu_1)] = \tilde{C}_1 L_S + \tilde{C}_2 [1 - U_{0,0}] + \tilde{C}_3 p^+ c_1 U_{0,0} + \tilde{C}_4 U_{0,0}$$

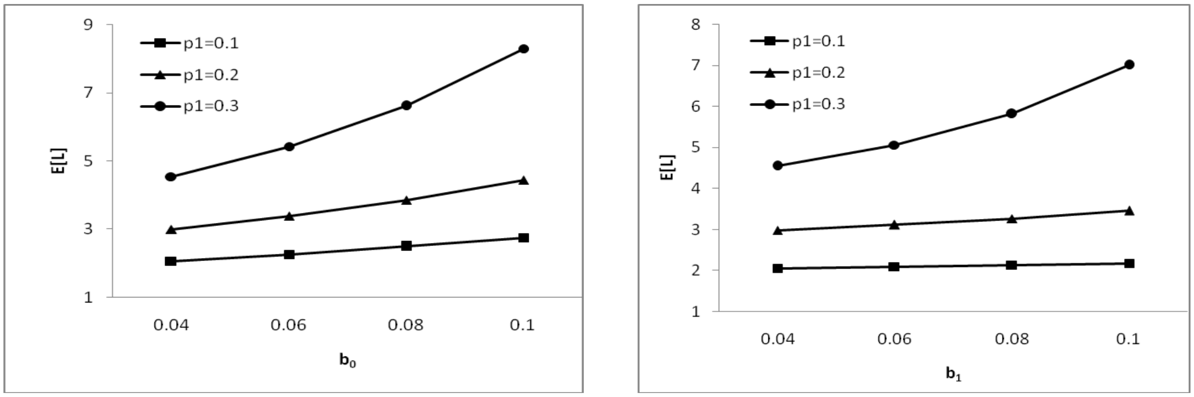
We try to use a well known heuristic approach "Direct Search Method" to get the joint optimal values ***(p_1^*, μ_1^*)*** which provide the least expected cost per unit time (cf. Jain and Upadhyaya, 2011). Also, the successive values of p_1 and μ_1 are put in the function defined for distinct cost sets such that one from the many parameters is fixed and the other variable varies. This action goes on unless we obtain the least total system cost $E[TC(p_1, \mu_1)]$ say E[TC*]. For the same, we opt p_1 and μ_1 to vary as [0.14, 0.15, 0.16] and [1.6, 1.8, 2, 2.2, 2.4, 2.6, 2.8], respectively. This activity is described as follows:

Step 1. Set p_1= 0.14. Determine $\mu_1^*(p_1) = \min\{\mu_1 > 0 \ / E[TC(p_1, \mu_1 + 0.2)] - E[TC(p_1, \mu_1)] > 0\}$ and compute $E[TC(p_1, \mu_1^*(p_1))]$.

Step 2. Compute $\mu_1^*(p_1+0.01)$ and $E[TC(p_1+0.01, \mu_1^*(p_1+0.01))]$.

Step 3. If $E[TC(p_1+0.01, \mu_1^*(p_1+0.01))] > E[TC(p_1, \mu_1^*(p_1))]$, STOP; the optimal values are **(p_1^*, μ_1^*)**=(p_1, $\mu_1^*(p_1)$). Otherwise, GOTO step 2.

In order to make the system cost effective, we are consideringthree sets of cost elements as shown in Table 1. The joint optimal values of p_1 and μ_1 and their respective minimum cost are displayed in Table 2. The default parameters for tables 1-2 are chosen as c_1=0.67; c_2=.0.89; λ=1.5; λ_0=0.5λ; λ_1=0.3λ; λ_2=0.2λ; q_1=q_2=0.5; b_0=b_1=0.04; $E[B_0]$=μ_1=0.8; $E[B_1]$=μ_2=0.2; $E[D_0]$=γ_1=0.25; $E[D_1]$=γ_2=0.2. Fig. 1(a-b) is constructed to show variation of essential and optional repair probabilities on average system size for different values of $p_{1.}$ From these figures, we observe that system size shows a linear trend for different values of these repair probabilities.



(a) **(b)**

Figure 1: Effect of failure rates(a) b_0 (b) b_1 on average system size.

6. CONCLUSIONS

This model fits best in telecommunication systems, computer systems, manufacturing system. We find probability generating function for the system size and queue length. Also, we find numerical results by using MATLAB. The future work in this field includes the concept of set up time taken by both server and repairmen before providing service and repair respectively.

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