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PERFORMANCE MODELING AND ANALYSIS OF SERVICE SYSTEMS WITH BATCH ARRIVALS

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ABSTRACT

This article studies the queueing model in which the customers are arriving in batches and getting served by the unknown number of servers i.e., we have the customers arriving in batch of random size at a service facility with unknown number of servers. The probability mass function functions of arriving batches assumed to be general and performance indices are obtained for Poisson, Binomial and Negative-binomial batch size. The expected shortest and longest service times have been predicted for specific service time distribution namely Weibull, Erlang, Hyper exponential, and Pareto distributions. Also, the numerical illustration is provided for the better understanding of the results.

Keywords: Queueing models, Batch arrival, General probability mass function, shortest and longest service times.

1. INTRODUCTION

Given the modern life scenarios, the study of queues either virtual or real (existential) has taken many different shapes to make things easier in all aspects of life. There is no exception in the fact that people want to utilize their time in the best possible way due to their hectic schedules. They have to complete pool of tasks within specified period of time and so waiting in the queue gives them the unpleasant experience. Therefore, the study of queues i.e., queueing theory becomes important. It provides base for many applied science branches like computer science, operations research and many of its results are used in advanced telecommunication systems, traffic engineering and reliability theory**.**

In this paper we discuss the shortest and longest service times for serving the customers arriving in batches at a service facility with unknown number of servers. We aimed at deriving the expected value of shortest and longest service times assuming Weibull, Erlang, Hyper-exponential, and Pareto distributions for batch service times while Poisson, Binomial and Negative-binomial probability mass functions for batch size. Exponential and uniform distributions for batch service time are considered by Scheuer (1973) for analyzing the shortest and longest service times for batches of jobs. We consider more general distributions in our model which have many practical applications in flexible manufacturing systems, congestion in road traffic, call centers etc.

2. LITERATURE REVIEW

In the classical queueing system, it is assumed that individual customers arrive at a service facility i.e., customers arrive one by one. However, in contrast, given the advanced real-world queueing situations the customers may arrive in groups or batches. In queueing theory, it is called as batch arrival and the queue so formed is called bulk queue. Such queues have a wide range of applications in production line and manufacturing systems. The research work done by Aissani (2000), Artalejo et al. (2004), Choudhary (2007) and Choudhary et. al. (2009) and books by Chaudhry et al. (1983), can provide the sufficient knowledge about the bulk queue (the queueing model with batch arrival).

Initially queueing systems with batch arrivals was introduced with Erlang's solution of the $M/E_K/1$ queue (Brockmeyer et.al. 1948) and stochastic modelling of queueing problems was developed by Kendal (1953), his theory of stochastic processes is based on method of embedded Markov chains. Neuts (1967) classified bulk queues with Poisson input. A perfect solution for a queueing problem with batch arrivals and correlated departures was obtained by Sharda (1973). Kulkarni (1986) derived the expressions for the expected waiting times in a multiclass batch arrival retrial queue.

There are many research articles in which the batch queues are studied with different phenomenon like retrial of the customers, baking behavior of the customer, unreliable server, vacation and working vacation. Abolnikov et. al. (1992) enlightened a class of bulk queueing systems with a compound Poisson input modulated by semi-Markov processes, multilevel control service time and queue length dependent, service delay discipline. Avrachenkov et. al. (2005) presented a research paper analyzing batch arrival processor-sharing with application to multi-level processor sharing scheduling. A batch arrival queueing system with threshold was discussed by Lee et. al. (1996). Chen (2006) offered a non- linear programming approach to derive the membership functions of the steady-state performance measures in bulk arrival queueing systems with varying batch size in that the arrival rate and service rate are fuzzy numbers. Haridass et al. (2008) studied bulk queue with unreliable server and single vacation, whereas single-server Poisson input queueing model, wherein arrivals of units are in bulk has been investigated by Singh et.al., (2014). Baruah et. al. (2014) studied the behavior of a batch arrival queueing system equipped with a single server providing general arbitrary service to customers with different service rates in two fluctuating modes of service. Ghimire et. al. (2014) presented mathematical models of M^b/M/1 bulk arrival queues.

In the present study, we considered the batch queue with unknown number of servers. There is very limited number of articles in which batch queue with unknown number of servers is studied. Shanbhag (1966) and Usha Kumari et al. (1998) considered an infinite server queue with batch arrivals, whereas Chakravarthy et. al. (2000) studied the model in which number of servers varies between lower and upper limit of the number of servers and Usha Kumari et al. (1998) studied the model in the continuous time.

Recently et al. (2018), Daw and Pender (2019), Chakravarthy et. al. (2020) and Choudhury et. al. (2020) considered bulk queue in their respective articles. Mohamed et al. (2018) provided a detailed analysis of bulk arrivals in queueing models, whereas a research paper on the distributions of infinite server queues with batch arrivals is presented by Daw et al. (2019). Chakravarthy (2020) too studied bulk queue with infinite upper bound on the size of the batch. They studied the model with batch service, which are generally distributed, and service times are independent of the batch size. They applied classical embedded Markov renewable process to understand the model and then applied continuous Markov chain. In addition, they used matrix-analytic method to find steady state solutions. Choudhury et. al. (2020) analyzed a model of batch arrival single server queue with random vacation policy. They considered general service and vacation time. and developed cost optimization policy in terms of average cost function to find locally optimal random vacation policy.

The paper studies the maximum and minimum service time taken by the server to server the batches of the customers. We have found the explicit results considering the combination of distributions for both batch size and service time. Therefore, this paper is organized as follows. In Section 2, we state the problem. We evaluate the minimum service time for the customers in section 3. Maximum service time of customers for various distributions are summarized in section 4. In section 5, numerical illustration is provided for the better understanding of the results and in section 6, the conclusive remarks are provided.

3. MODEL DESCRIPTION AND ASSUMPTIONS

This paper deals with the general distribution for both inter-arrival times and service-times. When needed we have mentioned the considered special cases. Following are the assumptions considered to study this model in the general context:

(i) The arrival of the customers is considered to be in groups or batches of random size N.

(ii) The number of servers can vary in the system as per requirement i.e., number of servers are unknown initially.

(iii) The service time of the i^{th} customer is denoted by T_i. The service times { T_i = 1, 2, ..., N} are mutually independent random variables, and their cumulative service time distribution is given by F(x).

(iv) The minimum and maximum time taken to serve all the customers in a batch are denoted by S and L, respectively. It is very evident that S and L will depend on the number of customers in a batch, the service time of each customer and the number of servers.

The service time of a batch will be minimum, if there are enough number of servers to serve each customer i.e., for each customer in the batch, there is a server available (i.e., service is in parallel). For $T_0 = 0$ and $N = n$, We have minimum service time as:

$$S = \max (T_0, T_1, \ldots\ldots\ldots, T_n) \quad (1)$$

As we are also interested to evaluate the maximum service time, it is very evident that service time will be maximum if there is only one server to serve all the customers present, therefore it serves the customers in turn (i.e., service is in series). Therefore, we have

$$U = \sum_{i=0}^{n} T_i \quad (2)$$

4. SHORTEST SERVICE TIME

Let F_s (x) denotes the cumulative distribution for S.
Then we have

$$\begin{aligned} F_S(x) &= p(S \le x) \\ &= \sum_{n=0}^{\infty} p\ [\ \max(T_0, T_1, \ldots\ldots\ldots, T_n) \le x \mid N = n]\ p_n \\ &= \sum_{n=0}^{\infty} \prod_{i=0}^{n} p\ (T_i \le x)\ p_n \\ &= \sum_{i=0}^{\infty} [F(x)]^n\ p_n \end{aligned} \quad (3)$$

On substituting p_n, the p.m.f. for different batch size distributions, F_S (x) can be obtained. E(S) can be evaluated using the formula:

$$E(S) = \int_0^{\infty} [1 - F_S(x)]\, dx \quad (4)$$

Case I: For Weibull service time distribution

$$F(x) = 1 - e^{-\lambda x^b}$$

Using equations (3) and (4), we have

$$E(S) = \begin{cases} \sum_{n=1}^{\infty} \frac{\beta^n}{\lfloor n} A_1 & \textit{For Poisson batch size} \\ \sum_{n=1}^{k} \binom{k}{n} A_1 & \textit{For Binomial batch size} \\ \sum_{n=1}^{\infty} (r)_n\, \rho^n A_1 & \textit{For Negative binomial batch size} \end{cases} \quad (5)$$

where,

$$(r)_n = \frac{\Gamma(r+n)}{\Gamma(r)\ \Gamma(n+1)} \quad \text{and} \quad A_1 = (-1)^{n-1} \frac{\Gamma(\frac{1}{b})}{b(\lambda n)^{1/b}}$$

Case II: For Erlang service time distribution

$$F(x) = 1 - \sum_{m=0}^{l-1} \frac{e^{-\lambda x} (\lambda x)^m}{\lfloor m}$$

We denote,

$$E \equiv E(\lambda x, l-1) = \sum_{m=0}^{l-1} \frac{e^{-\lambda x} (\lambda x)^m}{\lfloor m}$$

$$E(S) = \begin{cases} \sum_{n=1}^{\infty} \frac{\beta^n}{\lfloor n} A_2 & \textit{For Poisson batch size} \\ \sum_{n=1}^{k} \binom{k}{n} A_2 & \textit{For Binomial batch size} \\ \sum_{n=1}^{\infty} (r)_n\, \rho^n A_2 & \textit{For Negative binomial batch size} \end{cases} \quad (6)$$

where

$$A_2 = (-1)^{n-1} \int_0^{\infty} E^n dx$$

Since it is not easy to obtain the integral on R.H.S. of equation (6). The upper bound of $\int_0^{\infty} E^n dx$
Can be established by using the following preposition see Glynn (1987).

Preposition: Assume $\lambda x > 0, 0 \le l - 1 \le \lambda x$,
Then

$$E(\lambda x, l-1) \le e(\lambda x, l-1)(1 - \frac{l-1}{\lambda x})^{-1}$$

where

$$e(\lambda x, l-1) = \frac{e^{-\lambda x} (\lambda x)^m}{\lfloor m}, m = 0,1,2, \ldots$$

Now, using the above preposition the upper bound of $E(S)$ is given by

$$\int_0^\infty E^n dx \leq \frac{1}{(\lfloor l-1)^n} \sum_{r=0}^{\infty} \frac{(n)_r (l-1)^r}{\lambda (n)^{n(l-1)-r+1}} \lceil (n(l-1) - r + 1) \tag{7}$$

In particular, when $l = 2$, equation (6) reduces to

$$E(S) = \begin{cases} \sum_{n=1}^{\infty} \frac{\beta^n}{\lfloor n} A_3 & For\ Poisson\ batch\ size \\ \sum_{n=1}^{k} \binom{k}{n} A_3 & For\ Binomial\ batch\ size \\ \sum_{n=1}^{\infty} (r)_n\, \rho^n A_3 & For\ Negative\ binomial\ batch\ size \end{cases} \tag{8}$$

where

$$A_3 = (-1)^{n-1} \frac{\lceil (n+1, n)}{n^{n+1} \lambda}$$

Case III: For Hyper-exponential Distribution

$$F(x) = 1 - \sum_{m=1}^{l} \sigma_m\, e^{-\mu_m x}$$

Denote

$$H = \sum_{m=1}^{l} \sigma_m\, e^{-\mu_m x}$$

Then F(x) = 1 – H.

In this case, we have

$$E(S) = \begin{cases} \sum_{n=1}^{\infty} \frac{\beta^n}{\lfloor n} A_4 & For\ Poisson\ batch\ size \\ \sum_{n=1}^{k} \binom{k}{n} A_4 & For\ Binomial\ batch\ size \\ \sum_{n=1}^{\infty} (r)_n\, \rho^n A_4 & For\ Negative\ binomial\ batch\ size \end{cases} \tag{11}$$

where

$$A_4 = (-1)^{n-1} \int_0^\infty H^n dx$$

For l=2, equation (9) becomes

$$E(S) = \begin{cases} \sum_{n=1}^{\infty} \frac{\beta^n}{\lfloor n} A_5 & For\ Poisson\ batch\ size \\ \sum_{n=1}^{k} \binom{k}{n} A_5 & For\ Binomial\ batch\ size \\ \sum_{n=1}^{\infty} (r)_n\, \rho^n A_5 & For\ Negative\ binomial\ batch\ size \end{cases} \tag{12}$$

where

$$A_5 = \sum_{i=0}^{n} (-1)^{n+i-1} \binom{n}{i} \frac{\sigma_1^{n-i} \sigma_2^{i}}{\mu_2 i + \mu_1 (n-i)}$$

Case IV: For Pareto service time distribution

$$F(x) = 1 - \frac{1}{(1+bx)^{a+1}},\ a > 0, b > 0\ \&\ x > 0.$$

Using equation (3) and (4), we get

$$E(S) = \begin{cases} \sum_{n=1}^{\infty} \frac{\beta^n}{\lfloor n} A_6 & For\ Poisson\ batch\ size \\ \sum_{n=1}^{k} \binom{k}{n} A_6 & For\ Binomial\ batch\ size \\ \sum_{n=1}^{\infty} (r)_n\, \rho^n A_6 & For\ Negative\ binomial\ batch\ size \end{cases} \tag{13}$$

where

$$A_6 = (-1)^{n-1} \frac{1}{b[n(a+1)-1]}$$

5. LONGEST SERVICE TIME

Since batch size N and service times T_i are mutually independent, the maximum service time is given by:

$$E(L) = E(T)E(N) \tag{14}$$

For different distributions, the maximum service time can be summarized in the following table,

Table 1: Maximum service time

Batch size distribution / Service Time distribution	Poisson	Binomial	Negative Binomial
Weibull	$\frac{\Gamma(\frac{1}{b+1})}{\beta\,\lambda^{1/b}}$	$\frac{k\rho\,\Gamma(\frac{1}{b+1})}{\lambda^{1/b}}$	$\frac{r(1-p)\,\Gamma(\frac{1}{b+1})}{p\,\lambda^{1/b}}$
Erlang	$\frac{\beta k}{\lambda}$	$\frac{\beta pm}{\lambda}$	$\frac{r(1-p)m}{p\,\lambda}$
Hyper-exponential	$\frac{\beta}{\mu_m}$	$\frac{kp}{\mu_m}$	$\frac{r(1-p)m}{p\,\lambda\,\mu_m}$
Pareto	$\frac{1}{\beta ab}$	$\frac{kp}{ab}$	$\frac{r(1-p)}{pab}$

6. NUMERICAL RESULTS

In this section, numerical illustration is provided for the minimum service time for Weibull and pareto service time distribution each for Poisson, binomial, and negative binomial batch size. To find the numerical results, we have used MATLAB. For Weibull service time distribution, the numerical results are shown in Table 2-4, whereas for pareto service time distribution, the numerical results are represented with the help of 3-D surface graphs given by fig.1-3. For the computation of E(S) i.e., expected minimum service time, we have taken batch size n =5 (For Weibull service time distribution) and n=11(for pareto service time distribution) to get more insight into E(S).

Table 2: Effect of λ and β on the minimum service time $E(S)$ for Weibull service time and Poisson batch size

λ	β=0.2	β=0.4	β=0.6	β=0.8	β=1
0.2	0.06892	0.12024	0.15791	0.18517	0.20485
0.4	0.09746	0.17005	0.22332	0.26187	0.28970
0.6	0.11937	0.20827	0.27351	0.32073	0.35481
0.8	0.13784	0.24049	0.31583	0.37035	0.40970
1	0.15411	0.26887	0.35311	0.41406	0.45806

Table 3: Effect of λ and k on the minimum service time $E(S)$ for Weibull service time and Binomial batch size

λ	k=3	k=5	k=7	k=9	k=11
0.2	0.19396	0.16425	0.15062	0.14236	0.13665
0.4	0.27431	0.23229	0.21301	0.20133	0.19326
0.6	0.33596	0.28450	0.26088	0.24658	0.23670
0.8	0.38793	0.32851	0.30124	0.28473	0.27331
1	0.43372	0.36728	0.33680	0.31834	0.30557

Table 4: Effect of r, p and λ on the minimum service time $E(S)$ for Weibull service time and Negative-binomial batch size

(r, p)	**λ=0.2**	**λ=0.4**	**λ=0.6**	**λ=0.8**	**λ=1**
(1, 0.8)	0.07254	0.10259	0.12565	0.14509	0.16222
(1, 0.6)	0.17863	0.25263	0.30941	0.35727	0.39944
(1, 0.4)	4.36712	6.17604	7.56408	8.73424	9.76518
(1, 0.2)	741.12533	1048.10950	1283.66674	1482.25067	1657.20663
(2, 0.8)	0.12568	0.17774	0.21769	0.25137	0.28104
(2, 0.6)	0.51205	0.72415	0.88690	1.02410	1.14498
(2, 0.4)	26.98732	38.16583	46.74341	53.97464	60.34548
(2, 0.2)	4582.36906	6480.44847	7936.89603	9164.73812	10246.48872
(3, 0.8)	0.16605	0.23483	0.28761	0.33210	0.37130
(3, 0.6)	1.43414	2.02818	2.48400	2.86828	3.20684
(3, 0.4)	98.51753	139.32483	170.63737	197.03507	220.29190
(3, 0.2)	16403.87631	23198.58435	28412.34721	32807.75262	36680.18252
(4, 0.8)	0.20063	0.28374	0.34751	0.40127	0.44863
(4, 0.6)	3.68905	5.21710	6.38962	7.37810	8.24897
(4, 0.4)	272.52097	385.40285	472.02017	545.04194	609.37542
(4, 0.2)	44512.85534	62950.68373	77098.52705	89025.71069	99533.77042

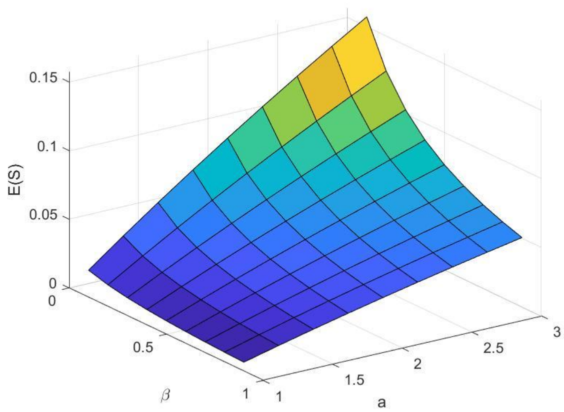

Fig. 1(i)

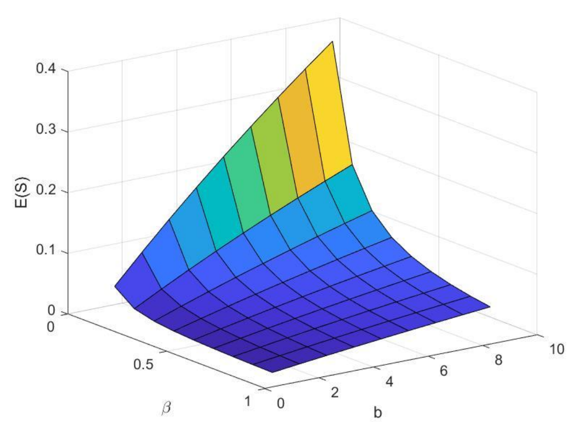

Fig. 1(ii)

Figure 1: For Pareto service time distribution and Poisson batch size

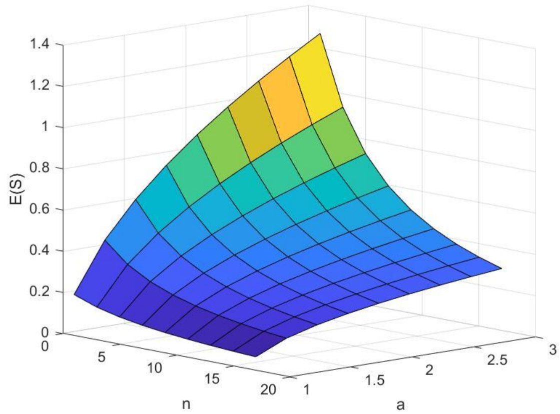

Fig. 2(i)

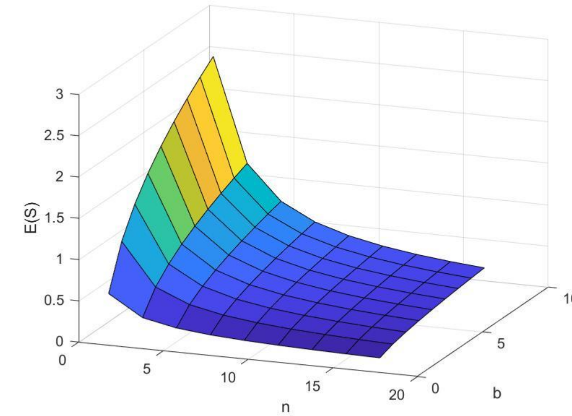

Fig. 2(ii)

Figure 2: For Pareto service time distribution and Binomial batch size

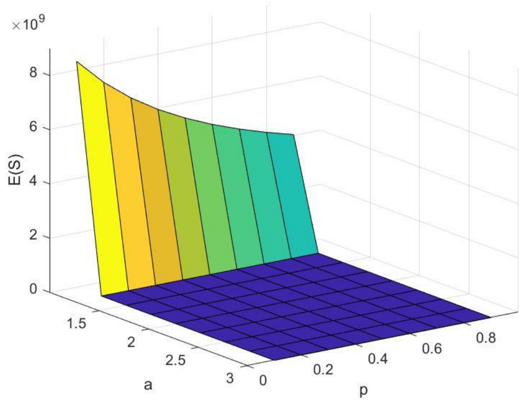

Figure 3(i)

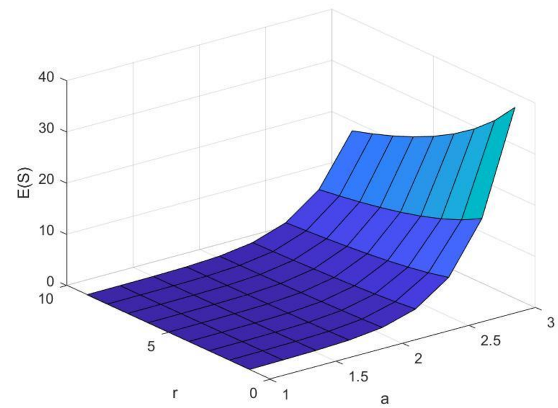

Figure 3(ii)

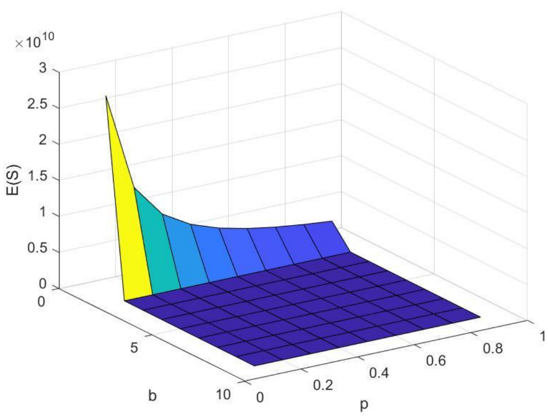

Figure 3(iii)

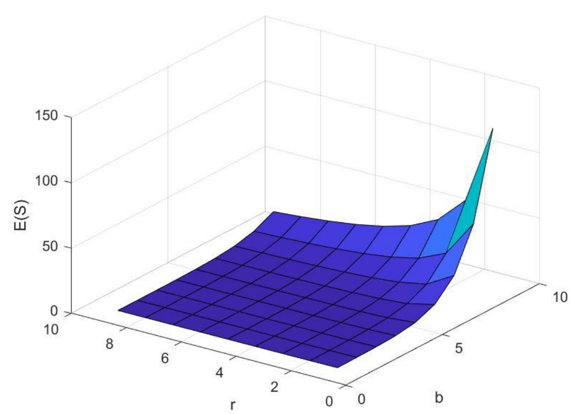

Figure 3(iv)

Figure 3: For Pareto service time distribution and Negative-Binomial batch size

7. CONCLUSION

In this study, we have provided expressions for Maximum and Minimum service times. To demonstrate the results obtained more efficiently, we have computed numerical results for Weibull and pareto service time distribution, each for Poisson, Binomial and Negative-binomial batch size distribution. For the computation of numerical results, MATLAB has been used. The future scope of this study is that it can be extended to the more general distribution for both arrival and service time distributions.

REFERENCES

Abolnikov, L., & Dshalalow, J.H. (1992). On a Multilevel Controlled Bulk Queueing System $M^X/G^{r,R}/1$. *Journal of applied Mathematics and Stochastic Analysis*, 5 (3), 237-260.

Aissani, A. (2000). An M^x/G/1 Retrial Queue with Exhaustive Vacations. *Journal of Statistics and Management Systems*, 3, 269-286.

Artalejo, J.R., & Atencia, I. (2004). On The Single Server Retrial Queue with Batch Arrivals. Sankhya: *The Indian Journal of Statistics*, 66, 140-158.

Avrachenkov, K., Ayesta, U., & Brown, P. (2005). Batch Arrival Processor-Sharing with Application to Multi-Level Processor-Sharing Scheduling. *Queueing System*, 50, 459-480.

Brockmeyer, E., Halstrom, H. L., & and Jensen, A. (1948). The Life and Works of AK Erlang. Transactions of the Danish Academy of Technical Sciences 2, The Copenhagen Telephone Company.

Baruah, M., Madan, K., & Eldabi, T. (2014). A batch Arrival Single Server Queue with Server Providing General Service in Two Fluctuating Modes and Reneging during Vacation and Breakdowns. *Journal of Probability and Statistics*, 1-12.

Chaudhry, M., & Templeton, J. (1983). A first course in bulk queues (1983), Wiley, New York.

Chakravarthy, S. R., & Dudin, A. N. (2000). A Batch Markovian Queue with a Variable Number of Servers and Group Services. *Matrix-analytic methods - theory and applications, eds. Latouche, G. & Taylor, P. (World Scientific Publishing Co. Singapore)*, 63–88.

Chakravarthy, S. R., Shruti & Rumyantsec, A. (2020). Analysis of a Queueing Model with Batch Markovian Arrival Process and General Distribution for Group Clearance. *Methodology and computing in Applied Probability*, 1-29.

Chen, S.P. (2006). A Bulk Arrival Queueing Model with Fuzzy Parameters and Varying Batch Sizes. *Applied Mathematical Modeling*, 30 (9), 920-929.

Choudhury, G. (2007). A Two-Phase Batch Arrival Retrial Queueing system with Bernoulli Vacation Schedule. *Applied Mathematics and Computation*, 188, 1455-1466.

Choudhury, M., Tedj, L., & Deka, K. (2010). A Batch Arrival Retrial Queueing System with Two Phases of Service and Service Interruption. *Computers & Mathematics with Applications*, 59, 437-450.

Choudhury, G., Kalita, P., & Dharmaraja, S. (2020). Analysis of a Model of Batch Arrival Single Server Queue with Random Vacation Policy. *Communications in Statistics-Theory and Methods*, 1-44.

Daw, A., & Pender, J. (2019). On the Distributions of Infinite Server Queues with Batch Arrivals.

Shanbhag, D. N. (1966). On Infinite Server Queues with Batch Arrivals. *Journal of Applied Probability*, 3 (1), 274–279.

Ghimire, S., Ghimire, R.P., & Thapa, G.B. (2014). Mathematical Models of M^b/M/1 Bulk Arrival Queueing System. *J. of Int. of Eng.*, 10 (1), 184-191.

Haridass, M., & Arumugananathan, R. (2008). Analysis of a Bulk Queues Unreliable Server and Single Vacation. *International Journal of Open Problems in Computer Science and Mathematics*, 1, 130-148.

Kendall, D.G. (1953). Stochastic Processes Occurring in The Theory of Queues and their Analysis by the Method of Imbedded, Markov Chains. *The Annals of Mathematical Statistics*, 24, 338-354.

Kulkarni, V.G. (1986). Expected Waiting Times in a Multi-class Batch Arrival Retrial Queue. *J. Appl. Prob.*, 144-154.

Lee, H.W., Yoo, S.H., & Lee, S.S. (1996). A Continuous Approximation for Batch Arrival Queues with Threshold. *Comp. & Pos. Res.*, 23, 299-308.

Mohamed, S.Y., & Karthikeyan, N. (2018). Analysis of Bulk Arrivals in Queueing Models. International *Research Journal of Multidisciplinary Studies*, 4 (4), 1-6.

Neuts, M.F. (1967). A General Class of Bulk Queues with Poisson input. *The Annals of Mathematical Statistics*, 38 (3), 759-770.

Sharda, (1973). A Queueing Problem with Batch Arrivals and Correlated Departures. Metrika, 20, 81-92.

Singh, C.J., Jain, M., & Kumar, B. (2014). Analysis of M^x/G/1 Queueing Model with Balking and Vacation. *Int. J. Oper. Res.*, 19 (2), 154-173.

Ushakumari, P.V., & Krishnamoorthy, A. (1998). On a Bulk Arrival Bulk Service Infinite Service Queue. *Stochastic Analysis and Applications*, 16 (3), 585-595.