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SUSTAINABLE INVENTORY MODEL OF DETERIORATING ITEMS WITH ADVANCE PAYMENT DISCOUNT POLICY UNDER INFLATIONARY ENVIRONMENT

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ABSTRACT

Numerous environmental issues that affect the modern era are caused by the release of carbon emissions from various sources, such as transportation and industry. The phenomenon of inflation is another important component of current inventory management systems. Keeping these issues in mind, we have proposed a sustainable inventory model of decaying items with partially backlogged shortages under an inflationary environment. This research aims to determine the optimal selling price and replenishment cycle that maximize a retailer's overall profit, considering that demand is influenced by both price and the level of carbon emission reduction. This research looks into an EOQ model, where the demand function not only influences the selling price but also positively contributes to carbon emission reductions. In this strategy, the supplier demands payment in advance on the purchase price while offering a discount based on the advanced payment decision. Two different cases of pre-payment ((1) pre-payment with an equal number of installments and (2) no payment in advance) have been performed with various numerical examples to validate the proposed model. Also, the concavity of the total profit for each case is shown by MATHEMATICA software. Finally, a sensitivity analysis is performed to gain valuable insights by examining the impact of different parameters on the model's outcomes.

Keywords: Advance payment; Inflation; price and carbon reduction dependent demand; deterioration; partially backlogged shortage.

1. INTRODUCTION

In the current market scenario, the concept of an advance payment discount facility has caught the great attention of various researchers. Competitive marketing techniques are the basis of existing business policy. Every businessperson or vendor faces great difficulty surviving on their own in the marketplace. Business owners makes use a variety of discount approaches such as reductions in prices, transportation discounts, quantity offers, trade discounts, discounts for payment in advance, etc., in order to attract clients and convert them into regular customers. Among such, quantity and trade discounts are customary in all businesses. The discount provided by the pre-payment option has just emerged as a new trend in market dynamics. The traditional economic order quantity (EOQ) approach anticipates payment to be made as soon as the goods are delivered. However, settlement in advance is frequently used, requiring the customer to make the purchase price before the goods are delivered. The buyer may be required to pay the entire purchasing price in advance or only a portion of it. Additionally, the pre-payment can be made over several periods. While the supplier will gain from the advance payment by reducing the possibility of cancellation, the supplier can also gain an advantage by giving customer-specific discounts. In the existing inventory system, another critical task to manage is deterioration. In reality, every product loses its usefulness and freshness after a given amount of time. The impact then persists over time. This is referred to as deterioration. It has an effect on inventory analysis as a result and should not be overlooked.

Concerns about the impact of consumer choices on industry growth have always existed. Numerous factors influence client preferences, reflected in their propensity to buy. Customer demand is typically responsive to product prices. Today's consumers, however, increasingly consider the producer's environmental performance and the product's level of greenness when making purchases. The expansion of this trend is occurring on a global scale, paralleled by the increasing consciousness among consumers about the significance of safeguarding the environment in response to the challenges posed by climate change. As a result, several manufacturers and merchants design green goods and encourage green business practices to draw in these customers. Regulations also play a vital role in this eco-innovation. To comply with new rules and customer demand, businesses are attempting to reduce carbon emissions from holding inventory, deterioration process, and transportation activities while implementing green technologies.

2. LITERATURE REVIEW

Current market conditions have led to a new trend of advance payment discounts. Generally, retailers have offered a pre-payment option to attract customers to purchase goods. The available literature has a lot of research on the discount facility. A few of them are briefly discussed here. Maiti et al. (2006) introduced a two-warehouse inventory system that deals for multi-items with a discount facility. Building on this work, Firoozi et al. (2013) conducted a study on optimizing a joint location inventory model with a quantity discount policy. In a similar vein, Mousavi et al. (2016) proposed a two-objective inventory model using discount facility under an inflationary context. Recent contributions by Shaikh et al. (2019), Shaikh et al. (2020), De (2020), and others have also focused on inventory models that incorporate price discount facilities.

The concept of advanced payment in business management was initially established by Zhang (1996), followed by subsequent research projects by renowned scholars. Maiti et al. (2009) examined the positive impact of the advance payment policy on an inventory system where demand is price-sensitive. Shaikh et al. (2019) examined a model for managing inventory in two warehouses, focusing on deteriorating items. They incorporated a pre-payment policy and employed particle swarm optimization techniques in their study. Similarly, Khan et al. (2020) investigated a model for a two-warehouse inventory system that deals with decaying items and takes pre payment into account. Expanding on this line of research, Taleizadeh et al. (2022) investigated an Economic Order Quantity (EOQ) model that incorporates environmental considerations. This study specifically looked at scenarios involving partial trade credit and partial backlogged shortages. Paul et al. (2022) introduced an inventory system that considers variable holding costs and retail investments with a specific focus on green operations.

Deterioration management is a crucial responsibility in the current inventory system. In actuality, every product loses its usability and freshness after a specified period. The first inventory model based on continuous deterioration was established by Ghare and Schrader (1963). Lee and Dye (2012) introduced an inventory system for decaying products in which the demand function is depends on stock. They investigate the impact of preservation investment on an inventory model in maximizing the profit. A preservation inventory model with shortage taking into account price-dependent demand was presented by Mishra et al. (2017). Inventory models for deteriorating items that are connected to preservation have been studied by Shaikh et al. (2019). Li et al. studied preservation technology for non-immediately deteriorating objects (2019). Mishra et al. (2021) developed an inventory system of deteriorating items by assuming preservation and green technologies. They discovered that the model with controllable deterioration and emission is more profitable than without them. Zhang et al. (2022) investigate the impact of carbon cap and trade policy on an integrated inventory model of deteriorating items.

Presently the status of the business environment is subjected to several changes, which foster the production sectors to promote sustainability in their domicile. Due to growing concerns about the capital discrepancy and depleting resources, businesses focus more on sustainability. Sustainable development is characterized as the ability to fulfill present needs while ensuring that future generations can meet their own needs without compromise. The World Commission on Environment and Development (WCED, 1987) offered an early definition of sustainability, describing it as "development that meets the needs of the present without jeopardizing the ability of future generations to meet their own needs." O'Connor (1993) makes one of the earliest allusions to sustainability, stating that the issue of "sustainability" is not simply an ecological (or even economic) debate but rather one of politics and ideology. In order to manage the environmental, social, and economic impacts of corporate operations, Font et al. (2008) introduced the concept of integrating sustainability into the supply chain, thereby defining it as a sustainable supply chain. The integration of sustainability principles into the supply chain has been extensively explored by researchers such as Ahi & Searcy (2013). In a related study, Mashud et al. (2021) developed an inventory approach that promotes sustainability by controlling carbon emissions.

Research Gap

Based on existing literature, several studies have explored the relationship between carbon reduction-dependent demand and the maximization of profits through lower carbon emissions. In light of these findings, the following research question arises: How can we construct a sustainable economic order quantity (SEOQ) model that incorporates an advance payment discount facility and carbon reduction-dependent demand simultaneously within an inflationary environment?

In response to this question, the proposed research develops a sustainable Economic Order Quantity model that integrates an advance payment discount facility in an inflationary environment. This model emphasizes investment in carbon emission reduction as a means to bridge the gap between economic and environmental sustainability. Furthermore, it recognizes the significance of time-varying holding costs in the dynamics of inventory models. Therefore, incorporating both inflation and variable holding costs into the system provides a realistic approach.

This study aims to develop a model for retailers that enable them to maximize their total average profit while minimizing environmental harm. By considering the interplay between advance payment discounts, carbon reduction-dependent demand, inflationary pressures, and variable holding costs, the proposed model aims to strike a balance between economic viability and environmental responsibility.

Table 1: Review of Previous Literature

References	**Price and carbon reduction-dependent demand**	**Variable holding cost**	**Inflation**	**Advance payment**	**Discount facility**	**Carbon Emission**	**Deterioration**
Hsu et al.(2010)							✓
Taleizadeh et al.(2014)				✓			✓
Mousavi et al. (2016)			✓	✓	✓		
Shaikh et al.(2017)			✓				✓
Shaikh et al.(2019)				✓	✓		✓
Lu et al.(2020)	✓					✓	✓
Mashud et al. (2020)		✓				✓	✓
Khan et al.(2020)		✓		✓	✓		✓
Shi et al.(2020)				✓	✓	✓	✓
Mishra et al.(2021)						✓	✓
Sultana et al.(2022)	✓			✓	✓	✓	
Present paper	✓	✓	✓	✓	✓	✓	✓

ASSUMPTIONS AND NOTATIONS

Assumptions

1) The holding cost is assumed to vary with time and is represented as $C_h = (g + ht)$, where $g > h > 0$. This accounts for the increasing cost of holding inventory over time.
2) The model is taken under an inflationary environment.
3) The retailer must pay advance payment as per the supplier's request. In return, the supplier offers a reduction in the price of the products supplied to the retailer.
4) The lead time for replenishing inventory is considered constant.
5) The replenishment rate is unlimited.

6) Partial backlogging of shortages is incorporated into the model with a fixed backlogging rate η .

7) The demand for the items depends on the selling price and the carbon reduction level, represented by the demand function D = α - βp + γR where α >β>0 , γ>0. Here, α is market potential, p is the selling price, β denotes the coefficient of price sensitivity, γ is the coefficient of low carbon preference, and R denotes the carbon reduction level.

Notation

Co**:** Ordering cost parameter
Cp**:** Per unit cost of purchasing items
C_h**:** Per unit holding cost / unit time
C_d**:** Deterioration cost / unit time
θ **:** Rate of deterioration (0< θ <<1)
η **:** Backlogging rate (0< η <1)
C_s**:** Cost for shortage / unit / unit time
C_l**:** Cost for lost sale / unit / unit time
Δ **:** Lead time
r**:** Rate of inflation
K**:** Percentage of discount based on advance payment
Q**:** Total ordered quantity
Ω**:** Initial inventory level
B**:** Backlogging quantity
ξ **:** Advance payment portion of the purchase cost
C_e**:** Cost for emission of carbon per unit distance
C_k**:** Cost for emission of carbon per unit products per unit distance
C_f**:** Consumption of fuel per ton of payload
F**:** Fixed transportation cost per trip
f**:** Consumption of fuel of an empty vehicle
N**:** Number of trips
l**:** Distance traveled
W**:** Weight of the product
ϕ **:** Rate of interest due to installment-based payment in advance
m**:** Number of installments
λ**:** Carbon emission reduction investment

Decision variables

t_1**:** Time when inventory level becomes zero
T**:** Cycle time
p**:** Selling price

3. MATHEMATICAL FORMULATION

With the abovementioned assumptions in mind, a sustainable inventory system is formulated, which incorporates carbon reduction-dependent demand. With the aforementioned assumptions in mind, a sustainable inventory system that integrates carbon reduction-dependent demand is developed. When payment-in-advance charges are included, the model is separated into two possibilities. In Case I, the advance payment is split into a certain number of equal payments, with the retailer receiving a discount based on the number of installments. In contrast, Case II does not involve any payment-in-advance cost, resulting in the retailer not receiving any discount.

3.1. Case I: With Discount for Advanced Payment in m Installments

In this scenario, the retailer operates their business with an initial supply of Ω units. The inventory level gradually declines as the demand and deterioration factors come into play, reaching zero at time t = t_1. From time t_1 to T, shortages arise with a consistent rate of backlogging denoted as η. Thus, when one cycle complete, the procedure is repeated to continue the business running. Before the products are delivered, the retailer pays a part of the purchasing cost (ξ) in m equal payments to get this inventory. The amount of each pre-payment is $\frac{\xi C_p \Omega}{m}$.The retailer must pay the remaining amount $\left(1-\xi\right)\Omega$ at the time of delivery. The facts mentioned above and the level of inventory pattern is shown in Figure 1.

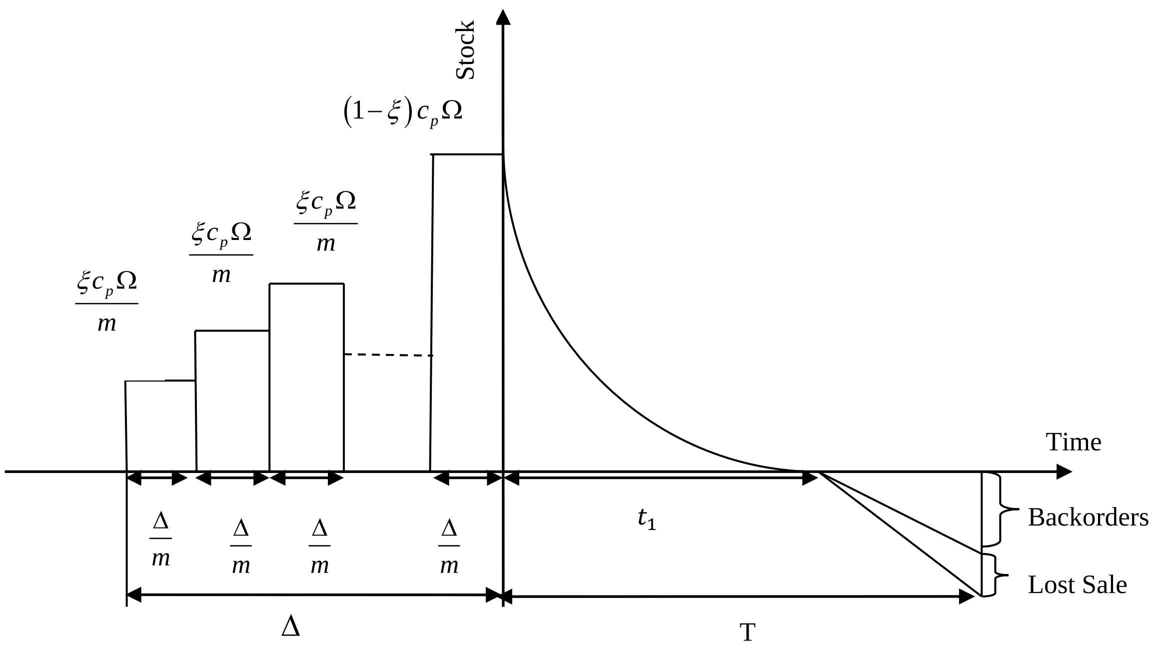


Figure 1: Representation of the level of inventory for Case I

Change in inventory level is given by

$$I'(t) + \theta I(t) = -D, \qquad 0 \le t \le t_1 \tag{1}$$

$$I'(t) = -\eta D, \qquad t_1 \le t \le T \tag{2}$$

With boundary conditions $I(0) = \Omega, I(t_1) = 0,$ and $I(T) = -B$

Using boundary condition $I(t_1) = 0$, the equation (1) gives the inventory level

$$I(t) = \frac{D}{\theta}\left(e^{\theta(t_1-t)} - 1\right) \tag{3}$$

Now the level of retailer's stock at time t is given by

$$I(t) = D\eta(t_1 - t) \tag{4}$$

The initial inventory level is

$$\Omega = \frac{D}{\theta}\left(e^{\theta t_1} - 1\right) \tag{5}$$

Now, using the boundary condition $I(t) = -B$ the maximum back-ordered quantity is

$$B = \eta D(T - t_1) \tag{6}$$

Now the inventory costs are

Ordering Cost per cycle

$$O.C. = C_o$$

Holding cost per cycle:

$$H.C. = \int_0^{t_1} (g + ht)\, e^{-rt} I(t) dt$$

$$= \frac{D}{\theta}\left((g + ht_1)\left(\frac{1}{r} - \frac{1}{(\theta + r)}\right)e^{-rt} - g\left(\frac{1}{r} - \frac{e^{\theta t_1}}{(\theta + r)}\right) - h\left(\frac{1}{(\theta + r)^2} - \frac{1}{r^2}\right)e^{-rt} - h\left(\frac{1}{r^2} - \frac{e^{\theta t_1}}{(\theta + r)^2}\right)\right)$$

Purchasing Cost per cycle

$$PC = C_p(\Omega + B)$$

Deterioration Cost per cycle:

$$DC = C_d \int_0^{t_1} \theta\, e^{-rt} I(t)dt$$

$$= C_d \left(\left(\frac{1}{r} - \frac{1}{(\theta + r)} \right) e^{-rt_1} - \left(\frac{1}{r} - \frac{e^{\theta t_1}}{(\theta + r)} \right) \right)$$

Shortage cost per cycle is:

$$SC = -C_s \int_{t_1}^{T} e^{-rt} I(t)\, dt$$

$$= -C_s D\eta \left(\frac{1}{r} (\, T - t_1) e^{-rT} + \frac{1}{r^2} (e^{-rT} - e^{-rt_1}) \right)$$

Lost cost per cycle is:

$$LSC = C_l (1 - \eta) \int_{t_1}^{T} D e^{-rt}\, dt$$

$$= -\frac{C_l D (1 - \eta)}{r} (e^{-rT} - e^{-rt_1})$$

Sales Revenue Cost is:

$$SR = p \int_0^{t_1} D\, e^{-rt} dt + pB$$

$$= \frac{Dp}{r} (1 - e^{-rt_1}) + pB$$

Installment Capital Cost is:

$$IC = \frac{\phi \xi C_p\, \Delta\, (\Omega + B)}{m^2} (m + (m - 1) + (m - 2) + \cdots + 2 + 1)$$

$$= \frac{(m + 1)}{2m} \phi \xi C_p\, \Delta\, (\Omega + B)$$

Discount on Purchase Costs: For additional installments, the supplier provides a lesser discount rate as follows:

$$\sigma = \frac{K}{n} where\ 0 \le K \le 100$$

Consequently, the overall discount provided by the supplier is

$$DCP = C_p \sigma (\Omega + B) = \frac{C_p K (\Omega + B)}{n}$$

The cost of transportation for each cycle is:

$$TC = N \left(F + \left(2alf + \frac{alC_f W (\Omega + B)}{N} \right) + \left(2lC_e + \frac{lC_k (\Omega + B)}{N} \right) \right)$$

Carbon Emission Reduction Cost is

$$RC = \lambda R^2$$

Now the overall profit per unit of time

$$TP(p, t_1, T) = \frac{1}{T} [SR - HC - OC - PC - DC - SC - LSC - IC - RC - TC + DCP]$$

$$= \frac{1}{T}\left[\frac{Dp}{r}(1-e^{-rt_1}) + pB - \frac{D}{\theta}\begin{pmatrix} (g+ht_1)\left(\frac{1}{r}-\frac{1}{(\theta+r)}\right)e^{-rt} - g\left(\frac{1}{r}-\frac{e^{\theta t_1}}{(\theta+r)}\right) - \\ h\left(\frac{1}{(\theta+r)^2}-\frac{1}{r^2}\right)e^{-rt} - h\left(\frac{1}{r^2}-\frac{e^{\theta t_1}}{(\theta+r)^2}\right) \end{pmatrix} - C_o - C_p(\Omega+B)\right.$$

$$- C_d\begin{pmatrix} \left(\frac{1}{r}-\frac{1}{(\theta+r)}\right)e^{-rt_1} - \\ \left(\frac{1}{r}-\frac{e^{\theta t_1}}{(\theta+r)}\right) \end{pmatrix} + C_s D\eta\begin{pmatrix} \frac{1}{r}(T-t_1)e^{-rT} + \\ \frac{1}{r^2}(e^{-rT}-e^{-rt_1}) \end{pmatrix} + \frac{C_l D(1-\eta)}{r}(e^{-rT}-e^{-rt_1})$$

$$\left. - \frac{(m+1)}{2m}\phi\xi C_p\,\Delta\,(\Omega+B) - \lambda R^2 - N\begin{pmatrix} F + \left(2alf + \frac{alC_f W(\Omega+B)}{N}\right) + \\ \left(2lC_e + \frac{lC_k(\Omega+B)}{N}\right) \end{pmatrix} + \frac{C_p K(\Omega+B)}{n}\right]$$

3.2. Case II: Without Advance Payment

The retailer does not provide an advance payment in this case. Therefore there is no discount or installment cost if the retailer makes no advance payment. Thus, in this instance, the retailer must complete the entire amount at the time of product delivery.

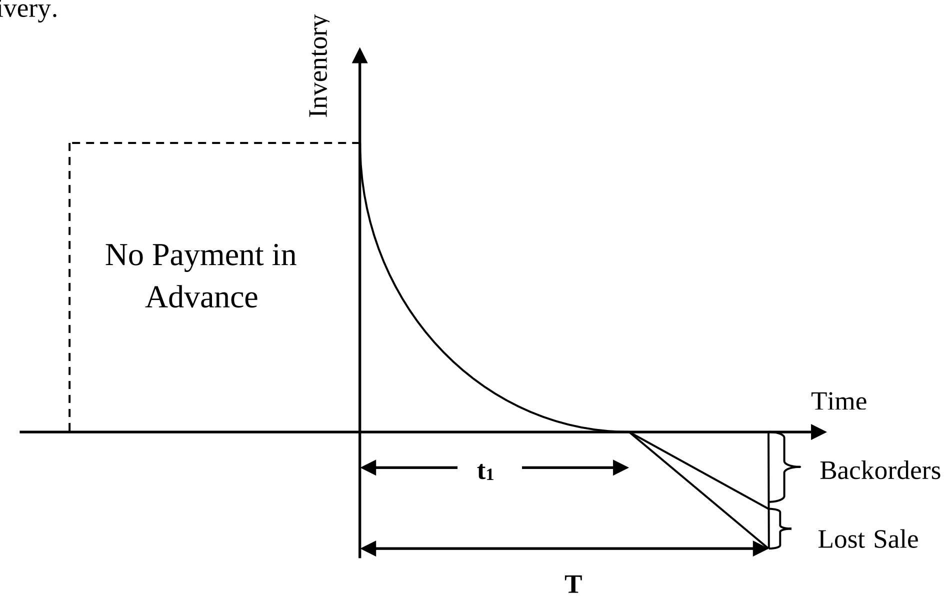


Figure 2: Graphical representation of the inventory level for Case II

Then the overall profit per unit time becomes

$$TP(p,t_1,T) = \frac{1}{T}[SR - HC - OC - PC - DC - SC - LSC - RC - TC]$$

$$= \frac{1}{T}\left[\frac{Dp}{r}(1-e^{-rt_1}) + pB - \frac{D}{\theta}\left(\begin{matrix}(g+ht_1)\left(\frac{1}{r}-\frac{1}{(\theta+r)}\right)e^{-rt} - g\left(\frac{1}{r}-\frac{e^{\theta t_1}}{(\theta+r)}\right) - \\ h\left(\frac{1}{(\theta+r)^2}-\frac{1}{r^2}\right)e^{-rt} - h\left(\frac{1}{r^2}-\frac{e^{\theta t_1}}{(\theta+r)^2}\right)\end{matrix}\right) - C_o - C_p(\Omega+B)\right.$$
$$- C_d\left(\begin{matrix}\left(\frac{1}{r}-\frac{1}{(\theta+r)}\right)e^{-rt_1} - \\ \left(\frac{1}{r}-\frac{e^{\theta t_1}}{(\theta+r)}\right)\end{matrix}\right) + C_s D\eta\left(\begin{matrix}\frac{1}{r}(T-t_1)e^{-rT} + \\ \frac{1}{r^2}(e^{-rT}-e^{-rt_1})\end{matrix}\right) + \frac{C_l D(1-\eta)}{r}(e^{-rT}-e^{-rt_1})$$
$$\left. - \lambda R^2 - N\left(\begin{matrix}F + \left(2alf + \frac{alC_f W(\Omega+B)}{N}\right) + \\ \left(2lC_e + \frac{lC_k(\Omega+B)}{N}\right)\end{matrix}\right)\right]$$

4. NUMERICAL ILLUSTRATION

Example 1. (Case II): In this example, costs associated with carbon emissions are taken into account without paying in advance. The following parameters are taken into account for numerical representation:
Ordering cost C_o = \$1000/cycle, coefficient of price sensitivity $\beta = 0.02$, coefficient of low carbon preference $\gamma = 10$, and reduction in carbon emission level R = 4, the demand combined with market potential $\alpha = 1.4$, investment for reduction in carbon emission λ = \$90. The holding cost parameters per unit time g = 2.7 & h = 0.1, deterioration rate $\theta = 0.08$ & deterioration cost parameter C_d = \$250/unit/month, shortage cost parameter C_s = \$100/month, opportunity cost parameter C_l = \$1/unit/month, purchasing cost parameter C_p = \$40/ unit, backlogging rate $\eta = 0.8$ and inflation rate r = 0.03. Further per unit distance carbon emission cost C_e = \$0.3/km, cost for per unit item per unit distance emission of carbon C_k = \$0.02/unit/km, fixed transportation cost F = \$300, Counting of trips N = 10, fuel amount a = \$0.1/liter, distance of travelling l = 20 km, fuel consumption of empty vehicle f =1 liter, product's weight W = 0.9 kg, payload fuel consumption C_q = 2 liter/ton.

Solution: We obtain the optimal solution for above example 1 using MATHEMATICA software
Selling price p = 1242.19/unit
Time (when inventory vanishes) t_1 = 8.32126 months
Replenishment time T = 13.3068 months
Total Profit TP = \$11437.9

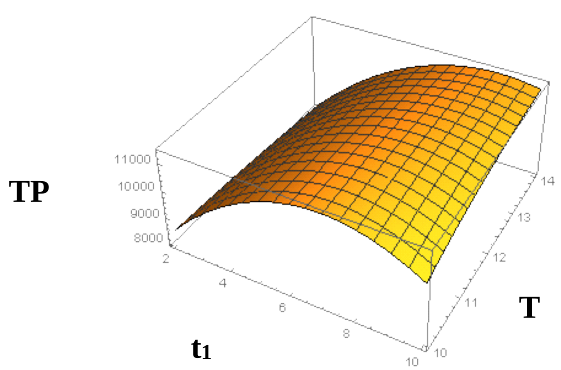
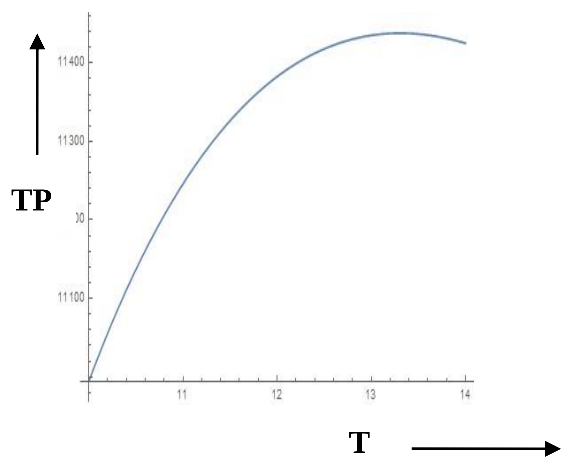


Figure 3: Concavity of overall profit (TP) w.r. to T and t_1 for Case II



Overall profit (TP) v/s cycle time T

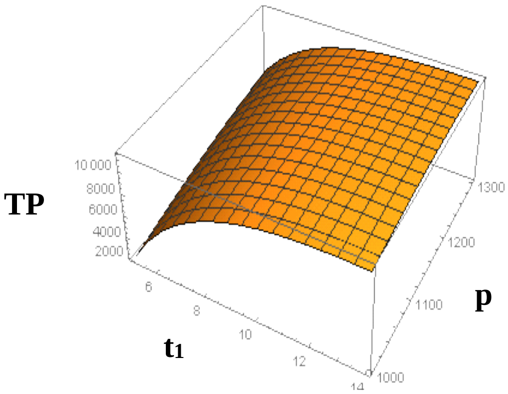
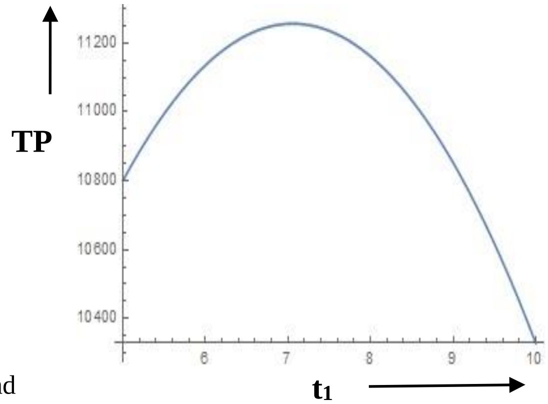


Figure 4: Concavity of overall profit (TP) w.r. to p and time t_1 for Case II



Overall profit (TP) v/s time t_1

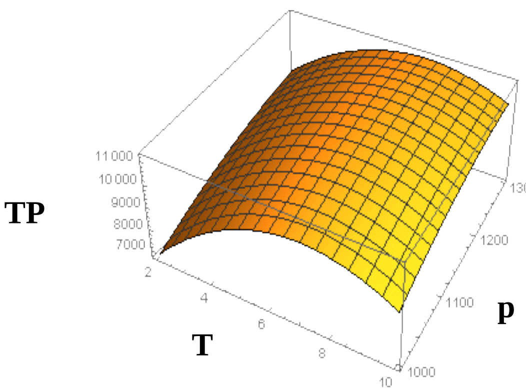
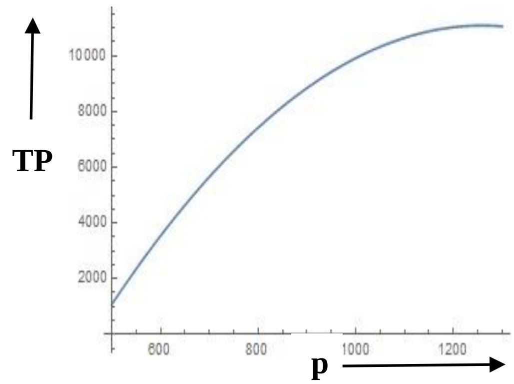


Figure 5: Concavity of overall profit (TP) w.r. to p and T for Case II



Overall profit (TP) v/s selling price p

Example 2. (Case I)In this case we take the same parameters are mentioned in above example 1 with additional parameters such as lead time L = 4 months, number of installments m = 10, advance fraction of purchase cost ξ = 0.1, discount rate for prepayment v = 5%, interest rate due to prepayment ϕ = 1.9
Solution: We obtain the optimal solution for above example 1 using MATHEMATICA software
Selling price p = 1131.25/unit
Time (when inventory vanishes) t_1 = 10.3739 months
Replenishment time T = 14.3271 months
Total Profit TP = $14620.7

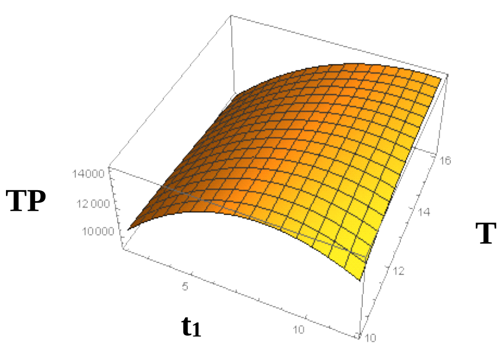
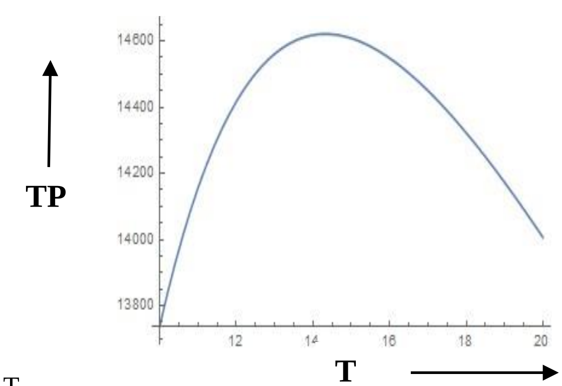


Figure 6: Concavity of overall profit (TP) w.r. to cycle time T and time t_1 for Case I



Overall profit (TP) v/s cycle time T

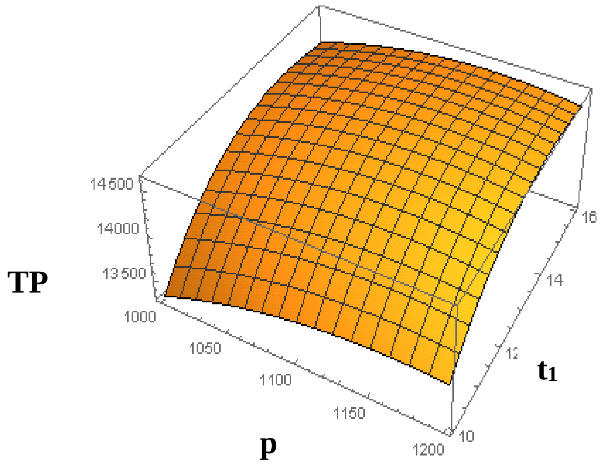
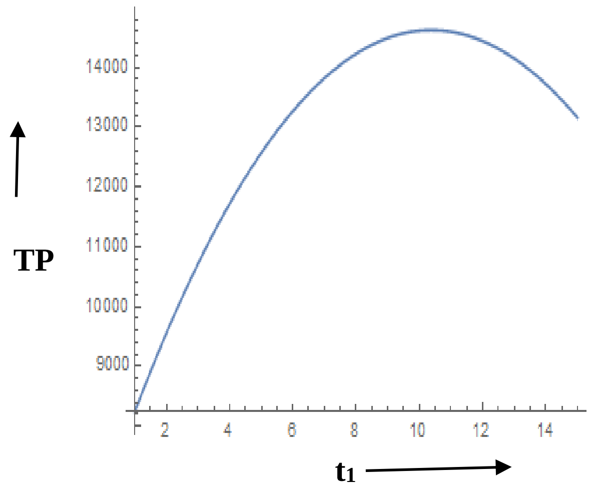


Figure 7: Concavity of overall profit (TP) w.r. to selling price p and time t_1 for Case I



Overall profit (TP) v/s time t_1

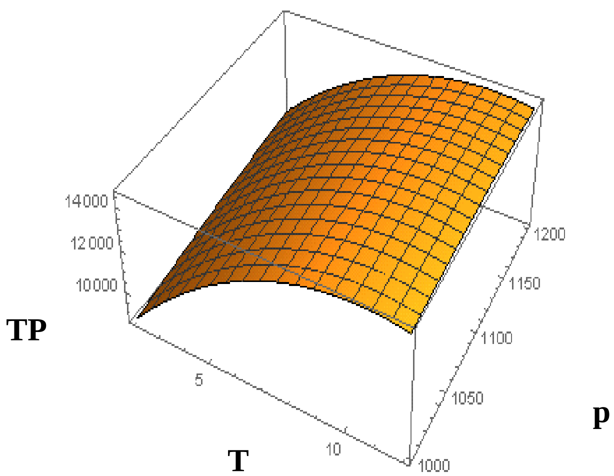
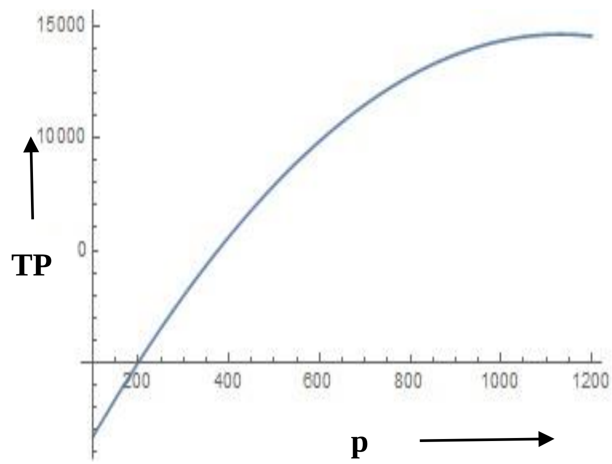


Figure 8: Concavity of overall profit (TP) w.r. to selling price p and cycle time T for Case I



Overall profit (TP) v/s selling price p

5. SENSITIVITY ANALYSIS

The present model depends on many parameters. In this section, the impact of these parameters has been studied with their sensitivity analysis. From numerical illustration, we have seen that the profit function is maximum for case I. So we perform the sensitivity analysis for the only case I (i.e., the case with advance payment) within the range +20% changes to -20%.

Table 2: Sensitivity Analysis w. r. to different parameters

Parameters	Change in %	Changed Value	Time t_1	Cycle Time T	Selling Price	Overall Profit (TP)
$\theta = 0.08$	-20	0.064	12.1635	16.7195	1132.76	14335.8
	-10	0.072	11.1679	15.4180	1132.04	14487.4
	+10	0.088	9.67491	13.3988	1130.41	14738.7
	+20	0.096	9.06437	12.5988	1129.56	14843.4

r=0.03	-20	0.024	13.0543	18.4646	1175.82	13271.0
	-10	0.027	11.6007	16.1926	1150.70	14021.3
	+10	0.033	9.34341	12.7776	1115.76	15110.0
	+20	0.036	8.46349	11.4750	1103.16	15516.4
R=4.0	-20	3.2	10.9418	15.1710	929.122	9008.53
	-10	3.6	10.6469	14.7333	1030.15	11645.3
	+10	4.4	10.1310	13.9471	1232.40	17936.0
	+20	4.8	9.90319	13.5891	1333.61	21592.1
n=10	-20	8	11.3540	14.9246	1093.87	15706.6
	-10	9	10.7613	14.5537	1115.65	15075.7
	+10	11	10.0962	14.1675	1143.56	14261.3
	+20	12	9.87964	14.0490	1153.55	13969.8
η=0.8	-20	0.64	10.8274	11.7304	1147.36	13967.8
	-10	0.72	10.6655	13.3183	1135.56	14218.6
	+10	0.88	10.0119	14.9549	1130.56	15114.6
	+20	0.96	9.60759	15.2868	1131.81	15642.9
g=2.7	-20	2.16	10.4873	14.3976	1129.39	14667.9
	-10	2.43	10.4321	14.3620	1130.32	14644.2
	+10	2.97	10.3246	14.2930	1132.16	14597.5
	+20	3.24	10.2722	14.2597	1133.06	14574.5
h=0.1	-20	0.08	9.92908	12.2862	1112.89	15388.6
	-10	0.09	9.92189	13.4601	1123.57	14937.1
	+10	0.11	10.8136	15.1696	1138.53	14326.9
	+20	0.12	11.2321	15.9922	1145.45	14052.5
C_p=40	-20	32	9.98157	14.1043	1148.79	14108.7
	-10	36	10.1734	14.2107	1140.11	14362.0
	+10	44	10.5970	14.4553	1122.18	14885.4
	+20	48	10.8267	14.5934	1113.11	15149.5
α=1.4	-20	1.12	10.3978	13.3886	1393.09	19119.1
	-10	1.26	10.3878	13.8963	1247.57	16614.7
	+10	1.54	10.6252	14.6982	1036.15	12996.5
	+20	1.68	10.8451	15.0217	956.946	11648.7
β	-20	0.016	9.77772	13.3886	1393.09	19119.1
	-10	0.018	10.0977	13.8963	1247.57	16614.7
	+10	0.022	10.6252	14.6982	1036.15	12996.5
	+20	0.024	10.8451	15.0217	956.946	11648.7
C_e=0.3	-20	0.24	10.3772	14.3258	1131.25	14621.2
	-10	0.27	10.3776	14.3265	1131.25	14621.0
	+10	0.33	10.3782	14.3278	1131.25	14620.5
	+20	0.36	10.3785	14.3284	1131.24	14620.2
λ=90	-20	72	10.3516	14.2739	1131.38	14640.9
	-10	81	10.3648	14.3005	1131.31	14630.8
	+10	99	10.3910	14.3537	1131.18	14610.7

	+20	108	10.4041	14.3802	1131.12	14600.7
L=4	-20	3.2	10.4471	14.3672	1128.33	14705.8
	-10	3.6	10.4123	14.3470	1129.79	14663.0
	+10	4.4	10.3438	14.3075	1132.70	14578.5
	+20	4.8	10.3102	14.2882	1134.14	14536.3
C_s=100	-20	80	10.2951	15.1913	1126.33	14753.3
	-10	90	10.3399	14.7109	1129.01	14681.1
	+10	110	10.4104	14.0132	1133.14	14569.6
	+20	120	10.4386	13.7514	1134.77	14525.6
C_l=1	-20	0.8	10.3777	14.3273	1131.24	14620.9
	-10	0.9	10.3778	14.3272	1131.25	14620.8
	+10	1.1	10.3780	14.3270	1131.25	14620.7
	+20	1.2	10.3781	14.3269	1131.25	14620.6
C_d=250	-20	200	11.2985	14.9406	1116.05	14996.4
	-10	225	10.8165	14.6137	1123.88	14805.7
	+10	275	10.0033	14.0908	1137.73	14454.0
	+20	300	9.66906	13.8866	1143.66	14298.6

6. OBSERVATIONS

Based on the sensitivity table, several helpful observations may be summarised.

- Greater values of the level of carbon emission reduction (R) have a positive effect on the retailer's total profit and selling price of the products but have an opposite impact on the time t_1 and cycle length (T).
- When the inflation rate (r) increases, the retailer's total profit (TP) increases while the selling price (p), time t_1, and cycle length (T) decrease. Fluctuations in the deterioration rate (θ) have been seen to have a similar kind of effect.
- Total profit and cycle length are directly impacted by the cost of the shortage (C_s) and the cost of the lost sale (C_l). The increased costs of those two factors result in a smaller profit, short cycle length, and higher selling price.
- The total profit (TP), time t_1, and cycle length all decrease as the number of installments (n) rises, while the selling price (p) increases. The same effect has been noted for lead time (L).

7. CONCLUSION

Global warming is influenced by factors such as carbon emissions and deterioration. Managers of greenhouses have emphasized reducing carbon emissions and have used low carbon consequences to do so. In this study, we examined a low carbon preference inventory system for decaying products with partially backlogged shortages in which demand is based on carbon emission reduction and the selling price of the products. This model is developed by considering the concept of an advance payment discount facility and variable holding cost under an inflationary environment which is a realistic situation.

The profit rises when the advance payment is made in fewer installments. This study demonstrates that the single payment scenario generates an enormous total profit and a lower selling price. Retailers should work more to reduce emissions in light of consumers' growing demand for environmentally friendly items. This study advises retailers to promote environmentally sustainable items in response to customers' increasing desires for low carbon to increase profits. To benefit from the discounts, retailers could reduce their prices and limit the number of payments to attract more customers.

Finally, the current study can be extended by considering hybrid payment policies, fuzzy-valued inventory cost, etc. Moreover, it can be developed by including some carbon emission regulations and taking a vendor-buyer system.

Here we have considered deterioration at a constant rate. So further this study can be extended by taking the time-dependent deterioration rate.

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