Global Journal of Modeling and Intelligent Computing (GJMIC)

Vol. 1, No. 2, July, 2021 ISSN: 2767-1917

INVENTORY MODEL FOR DETERIORATING ITEMS WITH FIXED SHELF-LIFE STOCK-DEPENDENT DEMAND AND PARTIAL BACKLOGGING WITH CAPACITY CONSTRAINTS

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ABSTRACT

In inventory management self life expiration date has an unique role. In the practice there are many goods and services such as food, medication remain safe and suitable for human consumption until it exist their shelf life. In this paper, we implemented a fixed shelf life of a deterministic inventory model for decaying items in two warehouses system with partial backlogging. The demand rate is considered stock dependent means that demand inclined by display of the stock level. In two warehouse system we are considering one own warehouse and second warehouse is on rented basis. The preservation facility is good in rented warehouse than own warehouse. Due to various preserve conditions, deterioration rate in two warehouses may be differs. In addition, backlogging rate is time dependent which is inversely proportional to the waiting time for the next cycle. The model is also justified by the numerical examples under two cases and also sensitivity analysis is carried out with various parameters by using MATLAB R16b.

Keywords: inventory model; shortage; stock-dependent demand rate; time-dependent partial backlogging rate; shelf life; capacity constraints.

1. INTRODUCTION

Due to growing in costs, decrement in resources level, undersized life cycle of the goods and various other reasons are responsible for incompetence in today's world wide market. Therefore, it is necessary to take various measures to make the business profitable. It is usually assumed that larger the display of the goods more goods will be purchased. From last few decades, many researchers led their attention towards the situation where demand rate is considered as stock dependent and results implies that the holding larger amount of goods will be cost-effective for the retailers. Gupta and Vrat (1986) established a deterministic inventory model with stock dependent consumption rate. Later et al. (1990) further studied two models with stock dependent demand rate, one with constant inflation rate and a (ther with exponential decay. Further, Mandal et al. (1989) studied an inventory model for deteriorating items with consumption rate as stock dependent. A model discussing various inventory problems of deteriorating items with shortages having stock dependent as demand rate by Datta et al. (1990). Karabi et al. (1996) purposed a model with two-component inventory demand rate with shortages. Three models i.e. without backorder, complete backorder and partial backorder was investigated by Padmanabhan et al. (1995). Further it was modified by Chung et al. (2000) discussing necessary and sufficient conditions for optimal solutions with different backorders. Some recents works was done by Zhou et al. (2005), Maiti et al. (2006), Goyal (2009), Yang et al. (2010), Kumar (2017), Palanivel (2016), Kumar et al. (2017), Pervin et al. (2019), Ullah et al. (2019).

In last few decades two warehouse inventory system was discussed by various researchers. Such a system was first proposed by Hartley (1976). He discussed about two warehouses i.e. own warehouse (OW) and rented warehouse (RW). It is assumed that holding cost for RW is always greater than OW. Therefore, the stock of RW was used to meet the demand of the customers until it drops to zero, then stock of OW is used. It manages the bulk purchase of the goods and retailer can get discount on it. Large pile of goods attracts more customers and proves to be more beneficial. Hence, models in inventory should be comprehensive to several warehouses.
With an assumption that holding cost of RW is larger than OW, Hartley (1976), introduced two warehouse inventory model. Further, Sarma (1987) broadened two warehouse inventory model with an infinite replenishment rate. Later,

Pakkala et al. (1992) introduced two warehouse system with deteriorating goods. Shah and Shah (1992) presented a deterministic model of inventory with two warehouse system and discussing the effects oqf permissible delay in payments. Later, Chung et al. (2004) presented a (ther model for (n- deteriorating goods with (shortages under permissible delay in payments. Zhou et al. (2005) established a (ther type of model having demand rate as stock dependent combination with two warehouse inventory system. Singhet et al. (2008) introduced shortages, inflation and time-value of money in one inventory model. Sett et al. (2012) presented an inventory model of two-warehouse with increasing demand and time varying deterioration. An inventory model with deteriorating items under permissible delay in payment and partial backlogging in two warehouse was discussed by Bhunia et al. (2014). An application of PSO in a two-warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies was presented by Bhunia et al. (2015). Two warehouse inventory policy with price dependent demand and deterioration under partial backlogging was established by Rastogi et al. (2017). Panda et al. (2019) discussed about credit policy approach in a two-warehouse inventory model for deteriorating items with price-and stock-dependent demand under partial backlogging. Some other authors, namely, Jaggi et al. (2015), Khurana (2015), Bhunia et al. (2016), Shabani et al. (2016) , Jaggi et al. (2017), Shaikh (2017), Pandaet et al. (2019), Mandal and (2019), Jaggi et al. (2019), Bishi et al.(2019), Yadav et al. (2019) done excellent work on it.

In this present paper, we develop the deterministic inventory model for decaying items in two warehouse system with shelf life under partial backlogging and demand rate of the inventory is inclined by quantity of stock level in display and we assumed backlogging rate during the stock out stage which inversely proportional to the for the future time of the next replenishment. This paper having following arrangement section 2 consists the assumptions and (tations and section 3 deliberated the mathematical model while section 4, explores the optimal solution of the model with numerical example to exemplify the model and the sensitivity analysis with the graphical representation by applying different parameters and followed by conclusion.

2. CIPHERS AND SUPPOSITION

The model of inventory with two warehouse system is developed with the following suppositions:

2.1 Ciphers

$I_r(t)$	Inventory level in rented warehouse(RW) for time t
$I_0(t)$	Inventory level in owed warehouse (OW) for time t
Z	Ordering cost for inventory, Rupees/ order
W	Capacity level of the own warehouse (OW)
S	Capacity level of the rented warehouse (RW)
H_1	Cost of holding, Rupees / unit time in rented warehouse (RW)
H_2	Cost of holding, Rupees / unit time in own warehouse (OW)
C	Cost of shortage, Rupee/ unit
L	Cost of lost sales, Rupee /per unit
D_1	Cost of deterioration, Rupee / per unit time in rented warehouse (RW)
D_2	Cost of deterioration, Rupee / per unit time in own warehouse (OW)
γ	Rate of deterioration in own warehouse(OW), value of γ lies in (0,1)
θ	Rate of deterioration in rented warehouse (RW), value of θ lies in (0,1)
t_1	Point of time for the inventory level goes down to zero in rented warehouse (RW)
t_2	Point of time for the inventory level goes down to zero in own warehouse (OW)
f_1	The time of fixed shelf life
T	Total sum of time of an inventory cycle, hence $T = t_1 + t_2$
$TC_1(t_1, T)$	Total sum of cost for the inventory cycle in case 1
$TC_2(t_1, T)$	Total sum of cost for the inventory cycle in case 2
TVC_1	Total sum of cost per unit time for an inventory cycle in case 1
TVC_2	Total sum of cost per unit time for an inventory cycle in case 2

2.2 Suppositions

1. The function used for demand rate D(t) is called function of instantaneous stock-level I(t) and considered as deterministic. It is defined as:
$$D(t)=\begin{cases}\alpha+\beta I(t), & 0\le t\le \mu\\ \alpha, & \mu\le t\le T\end{cases}$$ Where α, and β > 0.
2. The time horizon of the inventory system is infinite.
3. Both the warehouses i.e. own warehouse (OW) and rented warehouse (RW) has a predetermined capacity of W units and S units respectively.
4. After consumptions of the goods kept in rented (RW) then only the goods of own warehouse (OW) are consumed.
5. To guarantee the optimal solution exists, we assume that the maximum deteriorating quantity for times in OW, γW, is less than the demand date D, that, γW < D.
6. Shortages are allowed and demand is backlogged which is unsatisfied, and the shortages backordered is given by $(1+\delta(T-t))^{-1}$, where δ is a positive constant.

3. MATHEMATICAL FORMULATION

As we can see in Figure (a), we consider the following time intervals separately, $[0, t_1]$, $[t_1, t_2]$ and $[t_2, T]$. During the interval $[0, t_1]$, the inventory levels are positive at RW and OW. At RW, the inventory is depleted due to the combined effects of demand and deterioration. At OW, the inventory is only depleted by the effect of deterioration. Hence the inventory level at RW and OW are governed by the following differential equations

CASE 1: When $0 \le f_1 \le t_2$.

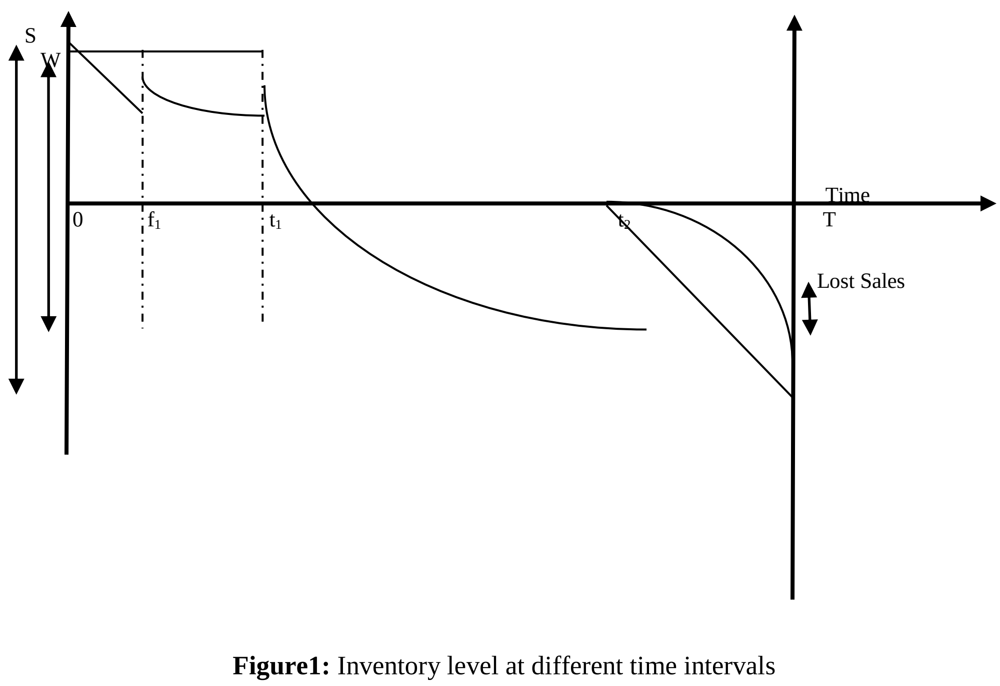


Figure1: Inventory level at different time intervals

Differential equations for rented warehouse for different time intervals

$$\frac{dI_r(t)}{dt}=-(\alpha+\beta I_r(t)) \qquad 0<t<f_1 \qquad (1)$$

$$\frac{dI_r(t)}{dt}=-(\alpha+\beta I_r(t))-\theta I_r(t) \qquad f_1<t<t_1 \qquad (2)$$

Solving equation (1) with boundary condition $I_r(0) = S$

$$I_r(t) = -\frac{\alpha}{\beta} + \left(\frac{\alpha}{\beta} + S\right)e^{-\beta t} \tag{3}$$

Solving equation (2) with boundary condition $I_r(t_1) = 0$

$$I_r(t) = \frac{\alpha}{\beta+\theta}(e^{(\beta+\theta)(t_1-t)} - 1) \tag{4}$$

Differential equations for owned warehouse for different time intervals

$$I_o(t) = W \qquad 0 < t < f_1 \tag{5}$$

$$\frac{dI_o(t)}{dt} = -\gamma I_o(t) \qquad f_1 < t < t_1 \tag{6}$$

Solving equation (6) with boundary condition $I_o(0) = W$

$$I_o(t) = We^{-\gamma t} \tag{7}$$

In between the interval $[t_1, t_2]$, the level of inventory in own warehouse (OW) is spent because of the effects of deterioration as well as demand. Hence, the level of inventory at own warehouse (OW) is given below by the differential equation given bellow:

$$\frac{dI_o(t)}{dt} = -(\alpha + \beta I_r(t)) - \gamma I_o(t) \qquad t_1 < t < t_2 \tag{8}$$

Solving equation (8) with boundary condition $I_o(t_2) = 0$

$$I_o(t) = \frac{\alpha}{\beta+\gamma}(e^{(\beta+\gamma)(t_1-t)} - 1) \tag{9}$$

Furthermore, at time t_2, the inventory level reaches zero in OW and shortages occur. During $[t_2,T]$, the inventory level only depend on demand, and some demand is lost while a fractio Moreover, at time t_b, the level of inventory go downs to zero in own warehouse (OW) and shortages occur. In between the time interval $[t_b,T_{ab}]$, the level of inventory depends on demand, and some fraction of demand got vanished but some fraction

$$\frac{1}{(1+\delta(T-t))} \tag{10}$$

is backlogged and where $t \in [t_2, T]$. The given below differential equation gives the level of inventory

$$\frac{dI_o(t)}{dt} = \frac{-\alpha}{1+\delta(T-t)} \tag{11}$$

With the use of boundary condition $I_o(t_2) = 0$, we get the inventory level as:

$$I_o(t) = -\frac{\alpha}{\delta}\left\{\log\left[1+\delta(T-t_2)\right] - \log\left[1+\delta(T-t)\right]\right\} \tag{12}$$

The total costs per cycle composed of certain constituents are given below:

1. Cost of ordering per cycle

$$OA = Z \tag{13}$$

2. Cost of holding cost per cycle in rented warehouse (RW)

$$HC_{RW} = H_1\left\{\int_0^{f_1} I_r(t)dt + \int_{f_1}^{t_1} I_r(t)dt\right\}$$

$$= H_1\left[\frac{\alpha}{(\beta+\theta)^2}\left\{e^{(t_1-f_1)(\beta+\theta)} - (\beta+\theta)(t_1-f_1) - 1\right\} - \frac{\alpha}{\beta}\left\{f_1 + \frac{S}{\alpha}e^{-\beta f_1} + \frac{e^{-\beta f_1}}{\beta}\right\}\right] \tag{14}$$

3. Cost of holding cost per cycle in owned warehouse (OW)

$$HC_{OW} = H_2\left\{\int_0^{f_1} I_o(t)dt + \int_{f_1}^{t_1} I_0(t)dt + \int_{t_1}^{t_2} I_0(t)dt\right\}$$

$$= -H_2\left[W\left(\frac{e^{-\gamma f_1}-e^{-\gamma t_1}}{\gamma}-f_1\right)+\frac{\alpha}{(\beta+\gamma)^2}(1-e^{(\beta+\gamma)(t_1-t_2)}+(\beta+\gamma)(t_1-t_2))\right] \tag{15}$$

4. Cost of shortage per cycle

$$SC = -C\left\{\int_{t_2}^{T} I_o(t)dt\right\} = \frac{C\alpha}{\delta^2}\{\delta(T-t_2)-\log[1+\delta(T-t_2)]\} \tag{16}$$

5. Lost sales per cycle

$$LS = L\alpha\int_{t_2}^{T}\left\{1-\frac{1}{[1+\delta(T-t)]}\right\}dt = \frac{L\alpha}{\delta}\{\delta(T-t_2)-\log[1+\delta(T-t_2)]\} \tag{17}$$

6. Cost of deterioration cost per cycle in rented warehouse (RW)

$$DC_{RW} = D_1\left\{\int_{f_1}^{t_1}\theta I_r(t)dt\right\} = \frac{-D_1\alpha\theta}{(\beta+\theta)^2}(1-e^{(\beta+\theta)(t_1-f_1)}+(\beta+\theta)(t_1-f_1)) \tag{18}$$

7. Cost of deterioration cost per cycle in owned warehouse (OW)

$$DC_{OW} = D_2\left\{\int_{f_1}^{t_1}\theta I_o(t)dt+\int_{t_1}^{t_2}\theta I_o(t)dt\right\} = D_2\left[\frac{W\theta}{-\gamma}(e^{-\gamma t_1}-e^{-\gamma f_1})+\frac{\alpha\theta}{(\beta+\gamma)^2}\left\{(e^{(\beta+\gamma)(t_2-t_1)}-1)-(\beta+\gamma)(t_2-t_1)\right\}\right] \tag{19}$$

Total Cost during time interval [0, T]

TC_1 (0, T) = OA+ $HC_{RW}+HC_{OW}+SC+LS+DC_{RW}+DC_{OW}$

$$\begin{aligned}
&= Z + = H_1\left[\frac{\alpha}{(\beta+\theta)^2}\left\{e^{(t_1-f_1)(\beta+\theta)}-(\beta+\theta)(t_1-f_1)-1\right\}-\frac{\alpha}{\beta}\left\{f_1+\frac{S}{\alpha}e^{-\beta f_1}+\frac{e^{-\beta f_1}}{\beta}\right\}\right] \\
&\quad -H_2\left[W\left(\frac{e^{-\gamma f_1}-e^{-\gamma t_1}}{\gamma}-f_1\right)+\frac{\alpha}{(\beta+\gamma)^2}(1-e^{(\beta+\gamma)(t_1-t_2)}+(\beta+\gamma)(t_1-t_2))\right] \\
&\quad +\frac{C\alpha}{\delta^2}\{\delta(T-t_2)-\log[1+\delta(T-t_2)]\}+\frac{L\alpha}{\delta}\{\delta(T-t_2)-\log[1+\delta(T-t_2)]\} \\
&\quad +\frac{-D_1\alpha\theta}{(\beta+\theta)^2}(1-e^{(\beta+\theta)(t_1-f_1)}+(\beta+\theta)(t_1-f_1)) \\
&\quad +D_2\left[\frac{W\theta}{-\gamma}(e^{-\gamma t_1}-e^{-\gamma f_1})+\frac{\alpha\theta}{(\beta+\gamma)^2}\left\{(e^{(\beta+\gamma)(t_2-t_1)}-1)-(\beta+\gamma)(t_2-t_1)\right\}\right]
\end{aligned} \tag{20}$$

Total Cost per unit time

TVC_1 (t_1, t_2) = $\dfrac{TC_1(t_1,t_2)}{T}$

$$= \frac{1}{T}\left[\begin{aligned}
&Z + H_1\left[\frac{\alpha}{(\beta+\theta)^2}\left\{e^{(t_1-f_1)(\beta+\theta)}-(\beta+\theta)(t_1-f_1)-1\right\}-\frac{\alpha}{\beta}\left\{f_1+\frac{S}{\alpha}e^{-\beta f_1}+\frac{e^{-\beta f_1}}{\beta}\right\}\right] \\
&-H_2\left[W\left(\frac{e^{-\gamma f_1}-e^{-\gamma t_1}}{\gamma}-f_1\right)+\frac{\alpha}{(\beta+\gamma)^2}(1-e^{(\beta+\gamma)(t_1-t_2)}+(\beta+\gamma)(t_1-t_2))\right] \\
&+\frac{C\alpha}{\delta^2}\{\delta(T-t_2)-\log[1+\delta(T-t_2)]\}+\frac{L\alpha}{\delta}\{\delta(T-t_2)-\log[1+\delta(T-t_2)]\} \\
&+\frac{-D_1\alpha\theta}{(\beta+\theta)^2}(1-e^{(\beta+\theta)(t_1-f_1)}+(\beta+\theta)(t_1-f_1)) \\
&+D_2\left[\frac{W\theta}{-\gamma}(e^{-\gamma t_1}-e^{-\gamma f_1})+\frac{\alpha\theta}{(\beta+\gamma)^2}\left\{(e^{(\beta+\gamma)(t_2-t_1)}-1)-(\beta+\gamma)(t_2-t_1)\right\}\right]
\end{aligned}\right] \tag{21}$$

CASE 2: When $t_1 \leq f_1 \leq t_2$.

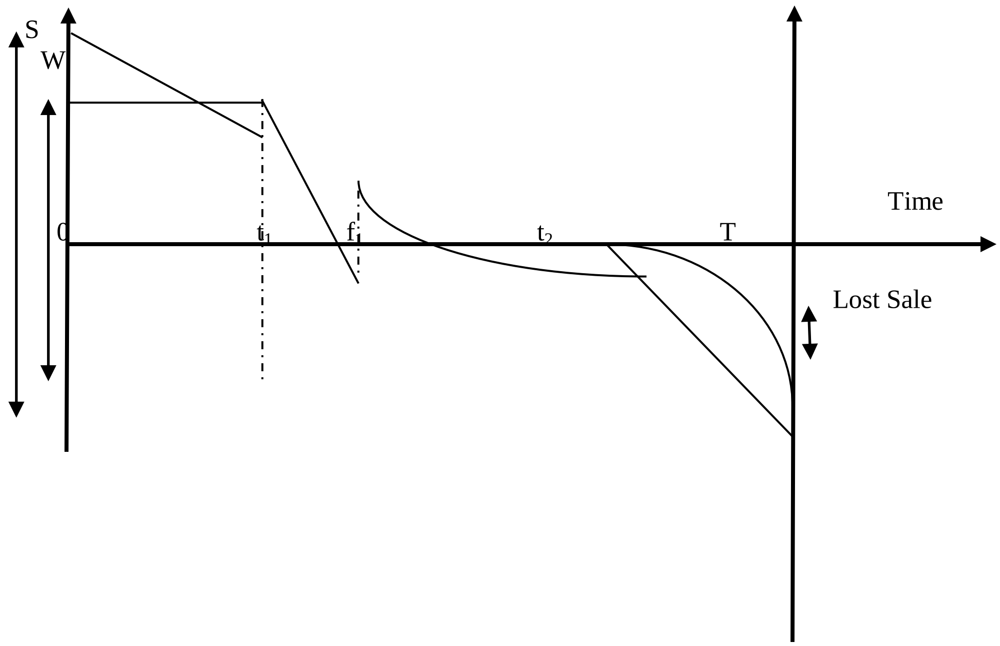


Figure 2: Inventory level at different time intervals

Differential equations for rented warehouse for different time intervals

$$\frac{dI_r(t)}{dt} = -(\alpha + \beta I_r(t)) \qquad 0 < t < t_1 \tag{22}$$

Solving equation (21) with boundary condition $I_r(0) = S$

$$I_r(t) = \frac{\alpha}{\beta}\left(\frac{1}{e^{\beta t}} - 1\right) + \frac{S}{e^{\beta t}} \tag{23}$$

Differential equations for owned warehouse for different time intervals

$$I_o(t) = W \qquad 0 < t < t_1 \tag{24}$$

$$\frac{dI_o(t)}{dt} = -(\alpha + \beta I_o(t)) \qquad t_1 < t < f_1 \tag{25}$$

$$\frac{dI_o(t)}{dt} = -(\alpha + \beta I_o(t)) - \gamma I_o(t) \qquad f_1 < t < t_2 \tag{26}$$

Solving differential equation (25) with boundary condition $I_o(t_1) = W$

$$I_o(t) = \frac{\alpha}{\beta}(e^{\beta(t_1 - t)} - 1) + We^{\beta(t_1 - t)} \tag{27}$$

Solving differential equation (26) with boundary condition $I_o(t_2) = 0$

$$I_o(t) = \frac{\alpha}{\beta + \gamma}(e^{(\beta+\gamma)(t_2 - t)} - 1) \tag{28}$$

Moreover, at time t_2, the level of inventory goes down to zero in own warehouse (OW) and shortages occur. In between the time interval $[t_2,T]$, the level of inventory depends on demand, and some fraction of demand got vanished but some fraction

$$\frac{1}{(1 + \delta(T - t))} \tag{29}$$

is backlogged and where $t \in [t_2, T]$. The given below differential equation gives the level of inventory.

$$\frac{dI_o(t)}{dt} = \frac{-\alpha}{1+\delta(T-t)} \tag{30}$$

Using the boundary condition $I_o(t_2) = 0$. We get the following level of inventory as:

$$I_o(t) = -\frac{\alpha}{\delta}\left[\log\left[1+\delta(T-t_2)\right] - \log\left[1+\delta(T-t)\right]\right] \tag{31}$$

The total cost per cycle composed of certain constituent is given below:

1. Ordering cost per cycle

$$OA = Z \tag{32}$$

2. Cost of holding cost per cycle in rented warehouse (RW)

$$HC_{RW} = H_1\left\{\int_0^{t_1} I_r(t)dt\right\} = H_1\left[\frac{\alpha+\beta W}{\beta^2}(1-e^{-\beta t_1}) - t_1\frac{\alpha}{\beta}\right] \tag{33}$$

3. Cost of holding cost per cycle in owned warehouse (OW)

$$HC_{OW} = H_2\left\{\int_0^{t_1} I_o(t)dt + \int_{t_1}^{f_1} I_0(t)dt + \int_{f_1}^{t_2} I_0(t)dt\right\}$$

$$= H_2\left[W + \frac{\alpha+\beta W}{\beta^2}\left(1-e^{\beta(t_1-f_1)}\right) - (f_1-t_1) + \frac{\alpha}{(\beta+\gamma)^2}(e^{(\beta+\gamma)(t_2-f_1)} - 1 - (\beta+\gamma)(t_2-f_1)\right] \tag{34}$$

4. Cost of shortage per cycle

$$SC = -C\left\{\int_{t_2}^{T} I_o(t)dt\right\} = \frac{C\alpha}{\delta^2}\left\{\delta(T-t_2) - \log[1+\delta(T-t_2)]\right\} \tag{35}$$

5. Lost sales per cycle

$$LS = L\alpha\int_{t_2}^{T}\left\{1 - \frac{1}{[1+\delta(T-t)]}\right\}dt = \frac{L\alpha}{\delta}\{\delta(T-t_2) - \log[1+\delta(T-t_2)]\} \tag{36}$$

6. Cost of deterioration per cycle in owned warehouse (OW)

$$DC_{OW} = D_2\left\{\int_{f_1}^{t_2} \theta I_o(t)dt\right\} = \frac{D_2\theta\alpha}{(\beta+\gamma)^2}(e^{(\beta+\gamma)(t_2+f_1)} - (\beta+\gamma)(t_2-f_1) - 1) \tag{37}$$

Total Cost during time interval [0, T]

$TC_2(0, T) = OA + HC_{RW} + HC_{OW} + SC + LS + DC_{OW}$

$$\begin{aligned} &= Z + H_1\left[\frac{\alpha+\beta W}{\beta^2}(1-e^{-\beta t_1}) - t_1\frac{\alpha}{\beta}\right] + H_2\left[\begin{array}{l} W + \frac{\alpha+\beta W}{\beta^2}\left(1-e^{\beta(t_1-f_1)}\right) - (f_1-t_1) \\ + \frac{\alpha}{(\beta+\gamma)^2}(e^{(\beta+\gamma)(t_2-f_1)} - 1 - (\beta+\gamma)(t_2-f_1) \end{array}\right] \\ &+ \frac{C\alpha}{\delta^2}\left\{\delta(T-t_2) - \log[1+\delta(T-t_2)]\right\} + \frac{L\alpha}{\delta}\{\delta(T-t_2) - \log[1+\delta(T-t_2)]\} \\ &+ \frac{D_2\theta\alpha}{(\beta+\gamma)^2}(e^{(\beta+\gamma)(t_2+f_1)} - (\beta+\gamma)(t_2-f_1) - 1) \end{aligned} \tag{38}$$

Total Cost per unit time

$$\mathrm{TVC}_2\,(t_1, t_2) = \frac{TC_2(t_1,t_2)}{T}$$

$$= \frac{1}{T}\left[\begin{array}{l} Z + H_1\left[\dfrac{\alpha+\beta W}{\beta^2}(1-e^{-\beta t_1}) - t_1\dfrac{\alpha}{\beta}\right] + H_2\left[\begin{array}{l} W + \dfrac{\alpha+\beta W}{\beta^2}\left(1-e^{\beta(t_1-f_1)}\right) - (f_1-t_1) \\ + \dfrac{\alpha}{(\beta+\gamma)^2}(e^{(\beta+\gamma)(t_2-f_1)} - 1 - (\beta+\gamma)(t_2-f_1)) \end{array}\right] \\ + \dfrac{C\alpha}{\delta^2}\left\{\delta(T-t_2) - \log[1+\delta(T-t_2)]\right\} + \dfrac{L\alpha}{\delta}\{\delta(T-t_2) - \log[1+\delta(T-t_2)]\} \\ + \dfrac{D_2\theta\alpha}{(\beta+\gamma)^2}(e^{(\beta+\gamma)(t_2+f_1)} - (\beta+\gamma)(t_2-f_1) - 1) \end{array}\right] \quad (39)$$

4. NUMERICAL EXAMPLES

For CASE 1: When $f_1 \le f_1 \le t_2$.

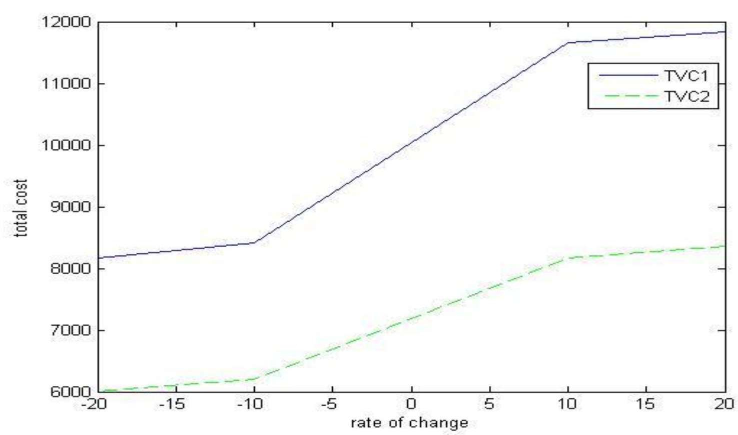
We assume the values as α = 570, β = 22.8, Z = 2000, H_2 = 12, W= 200, γ = 0.06, θ = 0.08, s = 30, D_1= D_2= 200, δ = 0.08, L = 15, t_1= 0.2228, t_2= 0.8818, f_1=0.1912, T=1.1047.after calculation the optimal value of TVC_1= 10087.87.

For CASE 2: When $t_1 \le f_1 \le t_2$.

We assume the values as α = 570, β = 22.8, Z = 2000, H_2 = 12, W= 200, γ = 0.06, θ = 0.08, s = 30, D_1= D_2= 200, δ = 0.08, L = 15, t_1= 0.1673, t_2= 0.8818, f_1= 0.3567, T=1.1047.after calculation the optimal value of TVC_2= 7182.31.

Table 1: Sensitive analysis of holding cost for owned warehouse (H_1)

Percentage change in parameter	TVC_1`	Percentage change in total cost	TVC_2	Percentage change in total cost
-20	8160.4721	-1.9273	6016.17	-1.1211
-10	8407.6217	-1.6802	6206.46	-0.9758
10	11662.1612	1.5742	8067.31	0.8849
20	11925.0761	1.8372	8360.70	1.1783

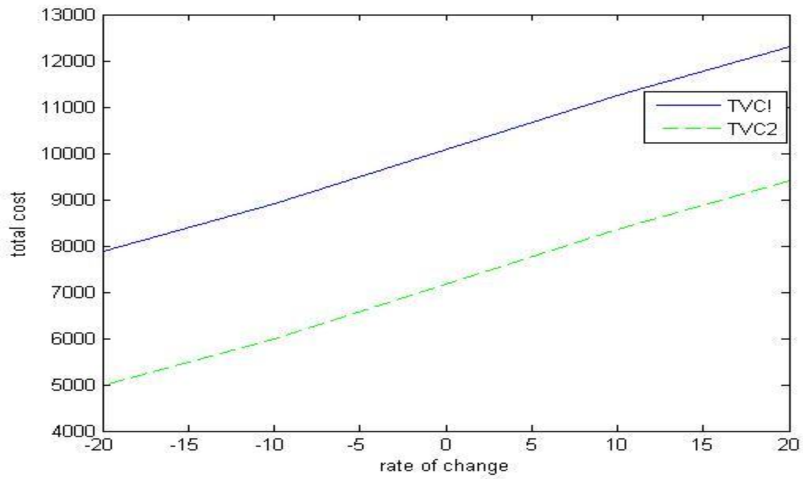


Observations:

I. From above table (1) it can be stated that cost per unit time changes (increases/decreases) with the change (increases/decreases) in holding cost of the own warehouse.
II. From the above graph it is clearly shown that the cost per unit time increase gradually in the above (both) cases with the increase in the value of holding cost of the own warehouse.
III. The value of TVC_1 is always larger than the value of TVC_2 for all the cases whether the holding cost increases or decreases.
IV. It's clearly observed that it is profitable to have larger extend of the fixed self-life.

Table 2: Sensitive analysis of holding cost for rented warehouse (H_2)

Percentage change in parameter	TVC_1	Percentage change in total cost	TVC_2	Percentage change in total cost
-20	7888.8120	-2.1999	4991.4768	-2.1908
-10	8908.7982	-1.1790	5978.5026	-1.2038
10	11250.5134	1.1626	8363.3129	1.1810
20	12290.2576	2.2023	9399.6814	2.2173



Observations:

I. From above table (2) it can be stated that the cost per unit time changes (increases/decreases) with the change (increases/decreases) in holding cost of the rented warehouse.
II. From the above graph it is clearly shown that the cost per unit time increase gradually in the above cases with the increase in the value of holding cost of the rented warehouse.
III. The value of TVC_1 is always greater than the value of TVC_2 for all the cases whether the value of holding cost of the rented warehouse increases or decreases.
IV. It's clearly observed that it is profitable to have larger extend of the fixed self-life.

5. CONCLUSION

In this paper, we considered an inventory system with two storage houses i.e. own warehouse (OW) and rented warehouse (RW), namely. We calculate the effect of shelf life and holding costs of both the warehouses and observe their effect on the inventory model. Shelf life is the recommendation of time period up to which the product remains acceptable under expected conditions of distribution, storage and display. Such k (wn time period is very much useful in consumption of commodities like fruits, vegetables, medicines, etc. From above numerical examples we can say that longer the shelf life period the cost decreases. The optimal cost hence obtained is the one we get when the shelf life time period in longer.

The effect of the modification in the value of cost of cost of own warehouse (OW) is shown in the table1 followed by the graph and its observation is made. Similarly, the effect of the modification in the value of holding cost of rented warehouse is shown in table2 followed by the graph and them its observations. By these observations we can easily say that with the increase of the holding costs of the warehouses the optimal cost increases and vice versa.

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