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A Review on Queuing Systems with Working Breakdown

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ABSTRACT

In this paper, different classes of queuing systems with working breakdown are surveyed in accordance with numerous important dimensions. Importance and basic characteristics of queuing system are discussed for some of the models. A comprehensive review has been given regarding the working breakdown process of server state. Behavior of server during the failure process, delay process, repair process, vacation state, busy state of various queues under different conditions are discussed for most of the papers with the help of various numerical and computational analysis done previously by many researchers since 2012. All the models are categorized separately for a better review and a detailed discussion is done in each subsection. The paper is completely focused on single server queueing system where working or partial breakdowns occur due to natural or human factor.

Keywords: Working Breakdown, Reliable server, Retrial queuing system, Markovian arrival process, M/G/1

1. INTRODUCTION

Queuing Theory, also commonly known as Waiting Line Models, is the most widely used technique in operations research which deals in turning real life scenarios into mathematical sense. Queuing system essentially occurs due to the arrival of unit at a server in order to get served. These arrivals are basically entities or people who require a service from the other entity and they have a tendency to leave the system after getting served. Therefore, there is a provision of service too. Arrivals and service are assumed to follow a certain distribution but they are not deterministic. Hence a queue is formed when there is not sufficient availability of resources for the satisfaction of demands of entities or when the server space is not enough, resulting in enforcement to wait for the service. For a better understanding one can refer to Kumar et al. (2015). They have described basic characteristics of queuing system along with a brief overview.

In classical queuing models, there goes a quite common assumption that the service station terminates completely whenever the server cease to work or breaks down for a lengthy or unpredictable period. A perfectly reliable server does not exist in reality and are ideal in nature. Hence, a new category of queuing models with breakdowns, called working breakdowns, was introduced by Kalidass et al. (2012). They considered a system which was subjected to failure during the operation, provided the server does not stop completely during the breakdown period but resumes to impart the service at a slower rate.

Breakdown can occur at any point and does not depend if the server is idle or busy. Hence, breakdown is of two forms, active breakdown and passive breakdown. If the breakdown occurs during the busy period, then it is called active breakdown. However, if the server fails in the idle state, then passive breakdown is said to occur. Considering the breakdown of server is an important factor else it would deteriorate the performance of the queuing system, hence reducing the efficiency.

Queuing systems with working breakdowns are widely used to analyse the efficiency measures of the system with the variation in breakdown rates. Hence, it enables the proxy server to issue service in the time of emergency during repair period by optimizing the system utilization. Therefore, it helps in sorting out congestion problems in real time systems by developing their designing and improvising their configuration. Queuing model is quite valuable and has a great potential, since it has many important applications in service systems and industries. Thus, many review papers are generated over the years based on queuing systems.

2. LITERATURE REVIEW

After a new notion of working breakdown was generated (Kalidass et al., 2012), the research was further developed by including the concept of server vacations, total breakdown, retrial customers, computation and optimization of working breakdown and many more, for example, Khalaf et al. (2012) presented a paper based on the assumption that when the main server resides in vacation period, customers are being served by the standby server that has been equipped in the system. Subramaniam et al. (2013) has studied a M/G/1 retrial queueing model with partial working breakdown. Li et al. (2013) considered an M/M/1 queueing model with infinite waiting area where they considered the working breakdown due to some natural or human influence. Kim et al. (2014) have analysed the queuing system with disasters and working breakdown, hence providing a scope to extend it to more complex situations. Yang et al. (2014) have used transient solutions for the development of system performance measures and sensitivity analysis is also performed. Liu et al. (2014) have dealt with bulk arrival queue with working breakdown. Liou (2015) examines the optimization of Markovian queue with infinite capacity and single server which cannot be relied upon, subject to reneging and balking customers along with working breakdown.

Li et al. (2017) considered a negative arrival along with normal arriving customers in an M/G/1 retrial queue. Jiang et al. (2017) have considered an environment dealing with disasters and working breakdown with multi-phase service along with elaborated analysis of performance measures. Ye et al. (2017) have analyzed MAP/M/1 queue with working breakdown and provided useful performance measures along with recursive formula for stationary sojourn time approximation. Bharathidass et al. (2018) have analysed a bulk service queue having server breakdown and repair along with derivation of state probabilities. Rajadurai (2018) have dealt with retrial queuing system having working breakdowns and working vacations. Jiang et al. (2018) have considered queuing model with one server in which working breakdown and delaying repair are observed under a specific policy. Ayyappan et al. (2018) have considered a non-Markovian queuing model which has one server with bulk arrival, focusing on various efficiency measures. Yang et al. (2018) have considered a queue with second optional service and has one server which is subject to working breakdown and repairs.

Ayyappan et al. (2018) have dealt with a non-Markovian model having breakdown along with a delay time under multiple vacation policy and consisting of a single server with bulk arrival. Deepa et al. (2018) have considered a M/M/1/N queue with working breakdowns and vacations. Ezeagu et al. (2018) has studied a finite M/M/1 model with working during breakdowns and recover policies during operation. Kalyanaraman et al. (2018) have dealt with a Markovian single server queuing system which has a tendency to alternate between busy state, breakdown state, working vacation state and repair state. Ammar et al. (2019) has investigated a M/G/1 retrial queue with priority customers along with disasters and working breakdown services. Chakravarthy et al. (2020) have dealt with MAP/PH/1 queue in which the server breaks down, take vacations and gets repaired. Yen et al. (2020) have analysed M/G/1 queues with N policy and their working breakdown. Rajadurai et al. (2020) analyzed an $M^x/G/1$ queue with retrials, preemptive priority and feedback along with balking of the customers due to disasters and working breakdowns.

The paper is organized as follows: In section 3, working breakdown processes of various queues under different conditions have been discussed with the help of results from various research papers published earlier. Section 4 concludes the paper.

3. PAPERS ON WORKING BREAKDOWN IN LITERATURE

3.1 M/M/1 Queue

Kalidass et al. (2012) considered a single queuing system in which arrivals follow Poisson distribution λ. They assumed that the server is subject to breakdown whenever it is in busy state. The service time distribution is exponentially distributed with a parameter $\mu_1 < \mu$. Time until the server breakdown is assumed as a parameter α and repair time is assumed as β, both being exponentially distributed. The inter arrival times, failure times, service times and repair times are independent of each other. They derived the stability condition for the system which is:

$$\rho = \frac{\lambda\alpha + \beta}{\alpha\mu_1 + \beta\mu} < 1$$

They also derived an equation for expected customer's number in the system E[X], i.e.:

$$E[X] = E[X_1] + E[X_2]$$

where $E[X_1]$ shows the customer's number in the system when it is in normal state and $E[X_2]$ represents the number of customers in the system when it is in defective state. Mean response time was found out to be $E[R] = \frac{E[X]}{\lambda}$

Following are the results of sensitivity analysis of performance measures done:

a) On increasing the value of arrival rate, the probability of server being idle decreases.
b) For better operation of system, value of μ_1 should be close to μ.
c) P_X and E[X] increases as λ increases.
d) E[X] decreases with increase in μ_1.

Li et al. (2013) discussed an M/M/1 queueing model with infinite waiting area. Two different states showed two different service rates, normal being μ whereas the slower rate being μ_0 that could be due to some natural or human factor. For the model they have evaluated the equilibrium threshold strategies for fully observable and unobservable cases that depends on customers viewpoint.

Yang et al. (2018) considered M/M/1 model with SOS. Here, FES has been issued to all the customers entering the system, while only a portion of them demanded SOS with probability ɣ ($0 \leq ɣ \leq 1$). It is assumed that FES and SOS are issued by the same server and has an exponential distribution with rates μ_{b_1} and μ_{b_2} respectively. Their breakdown rates are α_1 and α_2 respectively. In case of a failed server, repair times follow a distribution which is exponential in nature with a mean of $1/\beta_1$ and $1/\beta_2$ for FES and SOS respectively. During the period of breakdown, FES and SOS obey the exponential distribution with rates μ_{d_1} ($<\mu_{b_1}$) and μ_{d_2} ($<\mu_{b_2}$), respectively. For this model, they derived the stability condition which is:

$$\rho = \frac{\lambda[X + Y + ɣ(\mu_{d_1}M + \mu_{b_1}N)}{\mu_{d_1}X + \mu_{b_1}Y} < 1$$

where
X= $\alpha_2\mu_{b_1}\mu_{d_2}ɣ + \alpha_1\alpha_2\mu_{d_2} + \alpha_1\beta_2\mu_{b_2} + \alpha_1\mu_{b_2}\mu_{d_2}$
Y= $\beta_2\mu_{b_2}\mu_{d_1}ɣ + \alpha_2\beta_1\mu_{d_2} + \beta_1\beta_2\mu_{b_2} + \beta_1\mu_{b_2}\mu_{d_2}$
M= $\alpha_2\mu_{b_1}ɣ + \alpha_1\alpha_2 + \alpha_1\beta_2 + \alpha_1\mu_{b_2}$
N= $\beta_2\mu_{d_1}ɣ + \alpha_2\beta_1 + \beta_1\beta_2 + \beta_1\mu_{d_2}$

Kalyanaraman et al. (2018) have analyzed multiple working vacations and partial breakdown of M/M/1 queue using matrix geometric method. It is assumed that arrival rate becomes $\lambda_1(\lambda_1 < \lambda)$ during vacation. Server follows exponential distribution with rate μ_1 ($\mu_1 <$ μ) during vacation. A 3 X 3 matrix of the form $A = \begin{bmatrix} -\theta & \theta & 0 \\ 0 & -\alpha & \alpha \\ 0 & \beta & -\beta \end{bmatrix}$ is considered. Using a rate matrix R, they formed a matrix quadratic equation:

$$R^2A_2 + RA_1 + A_0 = 0 \quad \text{(i)}$$

According to the assumptions described above, they gave following stability condition for this particular model: $\rho = \frac{\lambda_2\alpha + \lambda\beta}{\mu\beta + \mu_2\alpha} < 1$

If ρ<1, equation (i) has the following minimal and a non-negative solution: $R = -A_0A_1^{-1} - R^2A_2A_1^{-1}$
They also derived following performance measures, provided p_0 is a stationery probability vector:

a) E(L), mean queue length = $p_0R(1 - R)^{-2}$e
b) E(L^2) = $p_0R(1 + R)(1 - R)^{-2}$e
c) Variance of queue length L, Var (L) = [$p_0R(1 + R)(1 - R)^{-2}$e] - [$p_0R(1 - R)^{-2}$e$]^2$
d) Probability of server being ideal = p_0e
e) During vacation period, mean queue length of server is given by $\sum_{i=0}^{\infty} ip_{i0}$

3.2 M/G/1 Queue

This system is an extended version of M/M/1 queue where arrivals are Markovian and general exponential distribution of service is a must. Subramaniam et al. (2013) has studied a M/G/1 queueing model with retrials and partial working breakdown. The service is provided with two different rates: normal rate μ_1 and minimized service rate μ_2 during partial breakdown. The model is solved using Direct Truncation Method. Kim et al. (2014) has analysed a M/G/1 model which has the following features:

- Arrival rate λ has a Poisson distribution.
- Inter arrival times possess exponential distribution.
- Customers are supposed to be served on FCFS basis.
- Inter arrival time of disasters have exponential distribution with a rate δ.

Occurrence of disaster results in failure of main server and the present customers are left with no option except to leave the system, followed by the beginning of repair process. During the repair process, customers are attended by proxy server, whose service rate is minimized than the main server.

They have derived the distribution of sojourn time and system size, performed cycle analysis and showed the relationship between sojourn time and system parameter. Hence, providing an extension to study the repairable queuing system.

Li et al. (2017) considered a negative arrival (G-queue) along with normal arriving customers in an M/G/1 retrial queue. The negative customers arriving in their system do not receive service and are in the system only for breakdowns during occupied period. Due to this sort of break, the service rate decreases and the server is sent for amendments. If the server is empty or under repair, the negative customers don't affect the system. The steady state analysis is carried out using supplementary variable technique which is further utilized in getting the performance measures. They then carried out the sensitivity analysis on the queue length and established the cost function to get the optimal service rate η and minimize the expected operational cost function.

Jiang et al. (2018) performed the computational analysis of a single server queue undergoing breakdown of working system and delay in repair system under a Bernoulli-scheduled-controlled policy. When the system breaks down, it either goes to a repair state with probability x or it works as a proxy server with a probability 1-x. When the system is in service period, it may go on repair if there are no customers in the system or when the working breakdown does not occur. Steady state distribution is obtained by using matrix analytic and spectral expansion method, which is further used for computation of performance measures.

Rajadurai (2018) has considered a retrial queuing system in which he has analysed steady state pgf of system size, using a variable technique. He also derived the system probabilities when the system is in different states (when the server is: idle, busy, lower service state, working vacation or working breakdown, failure), mean system size and orbit size, mean busy period and busy cycle. Let (P_i) be the probability that the server is unoccupied, (E_o) be the mean orbit size, (E_s) be the mean system size, (P_x) be the probability of server being unoccupied during retrial time, (P_b) regular busy, (P_w) working breakdown and working vacation, (P_y) server failure respectively. It was observed that E_o, E_s and P_x decreases with the increase in retrial rate. Whereas, P_i, P_b, P_w and P_y increases with the increase in retrial rate. E_o, E_s, P_i, P_w and P_y increases with increase in the value of rate of failure. P_x and P_b decrease with increase in failure rate value. P_i, P_b, P_w, E_o, E_s and P_y decreases with the increase in lower speed service rate.

On the other hand, Ammar et al. (2019) has examined a M/G/1 queue with retrials, priority customers along with disasters and working breakdown services. The priority customers pre-empty the usual customers which would make the ordinary customer to wait for the completion of his/her service. The disasters in the system would force the usual customers to leave the system thus failure of the initial server. At this period the server would be repaired and would provide service at a slower rate called working breakdown period. The steady state equation is solved using SVT and further the performance measures have been calculated. Further, a cost analysis and sensitivity analysis has been carried out for better understanding of the model.

Yen et al. (2020) have dealt with N policy M/G/1 queue with working breakdown. They have considered an important part regarding the working breakdown situation, that is, whenever the system undergoes a breakdown, customers keep on entering the system and form a queue and once the server is recovered, it serves the customer with a faster rate μ_1 ($\mu_1 > \mu$). Using PGF and supplementary variable techniques, they have derived the steady state probabilities for the given system. They even carried out sensitivity analysis for expected cost function.

3.3 G1/M/1 Queue

It is a dual of M/G/1 queuing system where general arrival process occurs but service times have exponential distribution. Jiang et al. (2017) have considered G1/M/1 queue in a multi-phase server environment undergoing disasters with working on breakdowns. In this model, there are two types of servers, main server (used in normal operative state) and proxy server (used during repairing period). During the occurrence of a disaster, initial server is sent to amendments and the service does not stop completely. They have obtained explicit computation solution for the performance measures, distribution of stationary queue length, stationary sojourn time distribution.

3.4 M/E_k/1 Queue

In Erlang queuing model M/E_k/1, customers mean arrival rate is λj (state dependent) and the service times follow an Erlang distribution with parameter k with mean service rate μ. Bharathidass et al. (2018) considered this model in which system state is represented as (a,b,c) where a (= 0,1,2,…) denotes customer's number available in the system, b (= 0) implies that unit is not in service, b (= 1,2,….,k) implies that unit is in b^{th} service state, c (= V,B,D, Rj) represents the position of server as :

$$m = \begin{cases} V, \text{ Working vacation state} \\ B, \text{ Turn – on and busy state} \\ D, \text{ Breakdown state} \\ Rj, \text{ under } j^{th} \text{ phase repair state} \end{cases}$$

The Erlang distribution has a total of k identical and independent distribution (iid) phases each having a mean $1/_{k\mu_0}$ during the working vacation period of server and $1/_{k\mu_1}$ during the busy state of server. Service is issued on the basis of FCFS discipline. System probabilities of steady state are defined as follows:

$P_{0,0,V}$: Probability of having no units in system and the server being on vacation
$P_{a,b,V}$: Probability of having a (≥0) units present in system when the server is rendering the b^{th} (1≤b≤k) phase of service while being in working vacation state.
$P_{a,b,B}$: Probability of having a (≥0) units present in system when the server is being rendered in b^{th} (1≤b≤k) phase of service and server is busy.
$P_{a,i,D}$: Probability of having a(≥ 0) units present in system m when the server is rendering b^{th} (1≤ b ≤ k) phase of service and the service provider is in break down state.
$P_{a,b,Rj}$: Probability of having a (≥ 0) units present in system when the server is rendering b^{th} (1≤ b ≤ k) phase of service and the server is under j^{th} (1≤ j ≤ y) phase of repair.

In this paper, they have derived steady state equations for the given queuing model. Probability generating functions are also derived in different states of server, these are:

$$P_V(z) = \sum_{b=1}^{k} X_b\,(z) = \sum_{a=1}^{\infty} \sum_{b=1}^{k} z^a\, P_{\mathrm{a,b,V}}$$

$$P_B(z) = \sum_{b=1}^{k} Y_b\,(z) = \sum_{a=1}^{\infty} \sum_{b=1}^{k} z^a\, P_{\mathrm{a,b,B}}$$

$$P_D(z) = \sum_{b=1}^{k} T_b\,(z) = \sum_{a=1}^{\infty} \sum_{b=1}^{k} z^a\, P_{\mathrm{a,b,D}}$$

$$P_{R_1}(z) = \sum_{b=1}^{k} N_{b1}\,(z) = \sum_{a=1}^{\infty} \sum_{b=1}^{k} z^a\, P_{\mathrm{a,b},R_1}$$

$$P_{R_j}(z) = \sum_{b=1}^{k} N_{bj}\,(z) = \sum_{a=1}^{\infty} \sum_{b=1}^{k} z^a\, P_{\mathrm{a,b},R_j}$$

where 2 ≤ j ≤ y.

They also discussed probability of various states of the system and performance measures of the queuing system.

3.5 M/M/1/N Queue

Yang et al. (2014) have considered M/M/1/N queue with working breakdowns and multiple vacations where system capacity is N (< ∞) and customer follows Poisson distribution with arrival rate λ and FCFS discipline. Service rate follows exponential distribution with service rate μ_b and it may breakdown randomly with breakdown rate α. This server has a repair time which is exponentially distributed with mean 1/β. During the repair times, the proxy server issues the service with a service rate μ_d $(< \mu_b)$. The server goes on a vacation if the system is empty with no customers in it. It returns from the vacation even if there is atleast one customer in the system, else remains in the vacation mode. Duration of vacations are exponentially distributed random variables with parameter ɣ.

They have modelled the given system using two-dimensional continuous-time Markov process $\{(J(t), A(t)); t \geq 0\}$ where A(t) denotes the numerical value of customers in system at time t and W(t) denotes the state of server at time t with:

J(t) =
- 0, in vacation period with normal server state
- 1, in normal busy period of server
- 2, in breakdown period of server
- 3, in breakdown vacation period of server

The state space of Markov process is: $S = \{(0,0), (3,0)\} \cup \{(x, y)|0 \leq x \leq 3, 1 \leq y \leq A\}$
Transient probabilities are defined as:

$$P_{x,y}(t) = Pr\{J(t) = x, A(t) = y\}, x = 0,3\ ;\ 1 \leq y \leq A$$

And

$$P_{x,y}(t) = Pr\{J(t) = x, A(t) = y\}, x = 1,2\ ;\ 1 \leq y \leq A$$

Therefore, they derived the state probabilities of system at time 0 :

$P_{0,0}(0) = 1$, $P_{x,y}(0) = 0$ ($0 \leq x \leq 3, 1 \leq y \leq A$) and $P_{3,0}(0) = 0$.

Deepa et al. (2018) did the steady state analysis and obtained steady state probabilities using computable matrix technique. They also developed various performance measures according to the given state. They have observed that the increase in value of λ for different values of μ led to the decrease in probability of server state being idle and system size increases with increase in arrival rate.

Ezeagu et al. (2018) has studied a finite M/M/1 model with working during breakdowns and recover policies during operation. They divided the recovery policies into two periods: *k*-threshold recovery and the startup recovery. The k-threshold recovery defines that the repair starts after $k \geq 1$ customer's number arrived in the system while the startup recovery happens once the system is unoccupied. Breakdown occurs if there is atleast one customer, and the broken server provides service at a minimized service rate. Further the model basic properties along with performance measures are calculated and the numerical illustration is done using graphs and tables.

3.6 MAP/M/1 and MAP/PH/1

MAP/M/1 is a queuing system which has Markovian arrival process (MAP) with single server queue. Ye et al. (2017) extended Kalidass et al. (2012) to MAP/M/1 with working breakdown and derived various important measures for the system. In this system, the server in normal conditions are exponentially distributed with parameter μ and the failure time is said to follow Poisson distribution with parameter α. When the server has gone for the repair, the repair time follows exponential distribution with parameter β. During the repair period, service rate becomes slower instead of pausing the service completely and it is said to follow an exponential distribution with parameter η (η<μ). Customers are served according to FCFS basis. Inter arrival times, failure times, service times during breakdown period and normal times, repair time are all independent of each other. They derived the stability condition for this queuing system. The QBD process $\{(X(t), K(t), Z(t)), t \geq 0\}$ is found to be stable if and only if, $\rho = \frac{\lambda(\alpha+\beta)}{\alpha\eta+\beta\mu} < 1$, where

$X(t)$: numerical value of customers in system at given time t
$Z(t)$: phase arrival at time t
$K(t)$: state of server at time t such that $J(t) = \begin{cases} 1, \ normal\ service\ state \\ 2, \ breakdown\ service\ state \end{cases}$

They have also discussed about the distribution of sojourn time using a recursive formula and have analyzed performance measures with the help of numerical examples.

Chakravarthy et al. (2020) did a qualitative study on MAP/PH/1 queuing model in steady state. It was done by analyzing various performance measures like sojourn time for various scenarios. However, it was observed that the dimension of problem increases when it was generalized to multi-server case.

Liou (2015) investigated an infinite capacity queue with Markovian property and having a single unreliable service system. This paper has contributed in permitting a direct sensitivity analysis of solution using traditional optimization approach.

3.7 Queues with Bulk Arrival

Finding the best suitable expressions to get the waiting time probabilities of the customers individually is a difficult process. Hence, computable approximations are carried out to find out these probabilities with the help of batch arrival queuing models, where the customer arrival takes place in batches and the service is given individually by a single server. Batch arrival of customers has a Poisson distribution. The service times and the customers present in the batches at a given time are iid positive random variables.

Khalaf et al. (2012) considered this queue with Bernoulli schedule vacation. In this model, server may go on a vacation of unspecified length after the service completion with probability p or stay in the system and issue the service with probability 1-p. It is assumed that the server may breakdown randomly and the repair process takes time to begin, once the breakdown occurs. In this work, authors have considered a case where a proxy server comes into action during the vacation state of main server. Service times, delay times and vacation times follow arbitrary distributions while exponential distribution is followed by the service time of standby server.

Liu et al. (2014) considered a $M^x/M/1$ queue along with working during breakdowns. They analysed a Markov chain with two dimensions and give its quasi upper triangle transition matrix. Using system stability condition, they derived the PGF of stationary queue length and further obtained its stochastic decomposition.

Ayyappan et al. (2018) in both his papers considered $M^x/G(a,b)/1$ model which has a non-Markovian batch arrival and has a general bulk service with single server queuing system. Since the server is unreliable, it is subject to breakdown at any instance. This results in suspension of server service and it is supposed to wait until the repair work is done. This waiting time is called waiting time which is assumed to follow general distribution. Ayyappan & Karpagam (2018) has resulted in derivation of queue size and performance measures of the system at any arbitrary time. While Ayyappan & Nirmala (2018) has given various performance indices along with PGF of queue size at arbitrary and departure epoch when the system is in steady state, using SVT.

Rajadurai et al. (2020) analyzed an $M^x/G/1$ queue with retrial, preemptive priority and feedback along with balking of the clients due to disasters and working breakdowns. Occurrence of disasters force all the customers to leave the system and cause failure in the main system. At this point of time comes in the substitute server which works as at lower service rate. The steady state pgf are calculated using SVT and some crucial performance measures are listed.

4. CONCLUSION

In this paper, we had a detailed discussion and a review about various queuing systems with working breakdown in different states. The concepts included in the article have quite valuable results. The derived results can help in improvising the reliability of the server, efficiency in machine replacement, flexible manufacturing systems and telecommunication systems. Sensitivity analyses of various performance systems are provided which can help in improvising the service of real time unreliable system. This article might help in dealing with emergencies that might occur during the repair period. This paper was mainly focused on single server queuing systems but it can be extended by reviewing queuing systems with multiple servers. Also, in this paper we considered the queuing systems with Markovian arrival process whereas the extension could be made on discrete time queueing models too.

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