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A MATHEMATICAL STUDY OF EPIDEMIC MODEL WITH REFERENCE TO COVID-19

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ABSTRACT

In order to aid in the planning of an effective strategy, we created a SAIUR model that forecasts the trajectory of the epidemic. We analyse local and global durability with regard to both the fundamental R_0 and the infection-free steadiness point. Additionally, the SAIUR model demonstrated illness persistence for so that if the epidemic dies out, an endemic equilibrium will be created. We show how our SAIUR model's sensitivity analysis may be used to determine the factors that are most crucial for disease transmission. With the use of an assumed and estimated parameter, we determine the R_0's sensitivity indices, and the simulation outcomes of our model indicate that the disease transference rate is more efficient in reducing R_0.

Keywords: Covid-19, Asymptote & Suggestive case, Stability analysis

1. INTRODUCTION

An epidemic is an illness that spread rapidly and effects a lot of people in a short period of time. It has a lengthy history and a huge selection of models for different diseases. Stress is often expected to rise during infectious disease outbreaks that are brought on by a variety of variables related to modifications in the host population's bionomics. The definition an epidemic is when a disease spreads rapidly from person to person in an area where the sickness cannot be completely eradicated. In view of this, WHO has revealed COVID-19 as a pandemic. Epidemiology is a branch of science that focuses on the factors that affect the incidence, spread, and management of infectious diseases in human populations.

2. LITERATURE REVIEW

In a stochastic SIS model related to respiratory disease with random diffusion of air pollutants (Boukanjine, 2019) developed an epidemic model with the double hypothesis and combined two different transmission mechanisms of (Susceptible Infectious Susceptible) and (Susceptible Infectious Removed) with two different non-linear incidence rates. A speculative SIR epidemic model with logistic growth was created by (Liu and Jiang 2020). The median daily R_0 in Wuhan decreased by 2.35 in the week prior to the travel limitation to 1.05. (Kucharski et al.,2020) researched this phenomenon. The ongoing COVID-19 global crisis report of all confirmed cases recovered cases, and deaths in India was researched by (Piu Samui et al.,2020). By comparing the new ay out features of COVID-19 with those of MERS and SARS and taking the rising rate and inhibition constant of infectious diseases into account, (Liang 2020) analysed the spread rules of the three pneumonias are MERS, COVID-19, and SARS. The nonlinear fitting was employed to determine the parameters for the three models of coronavirus transmission growth. The COVID-19 doubling cycle was two to three days, and the parametric analysis revealed that the rising rate was roughly double that of MERS and SARS. This suggests that the number of without human intervention, the COVID-19 patient would double in 2-3 days. (Rabiah et al.,2020) Media rumours may have escalated as a result of the MERS-COV outbreak in 2014, which was linked to a high level of social fear in the afflicted nations. Because they were most at danger for infection, the medical community was the most upset. This report was the only one to investigate how the MERS-COV epidemic affected medical students' psychological discomfort throughout this outbreak and what factors contributed to it. After the lockdown period, (Timothy et al., 2021) evaluated the data on international travel between nations and assessed COVID-19 transmission under various pandemic conditions (May-Sep). In order to determine the precise age for vaccination, (Brody H. Foy 2021) examined mathematical modelling and compared COVID-19 vaccine techniques in India. They also employed an age-structured and enlarged SEIR model. Children's mental health amid lockdown, seclusion, and quarantine was examined in march (Yunhe Wang 2021) studied the negative impacts of quarantine and isolation in the COVID-19 pandemic due to physiological and social impacts. (WHO 2020) report of COVID-19 developed a report of probable or confirmed cases of COVID-19 and also published the report of Health Care Worker (HCW) infection and died out. In the midst of this outbreak, many people lost their financial stability. We expand the model created by (Khajanchi et al., 2021). This study's primary goals are to look over the stability analysis of endemic equilibrium (EE) and disease-free equilibrium (DFE), as well as to research key factors influencing coronavirus transmission and attempt to create efficient methods of disease management. We presume that the recovered human beings will never re-join the susceptible class. Therefore, we investigate the DFE and EE through stability analysis.

The remainder of the essay is structured as follows: We describe the SAIUR model and ascertain the fundamental R_0 in section 2. We covered a few qualitative aspects of the SAIUR model in section 3. Section 4 discusses model analysis, which includes stability analyses of EE and DFE. In the same part, we also analysed the disease persistence with regards to R_0 and calculated local and global stability analyses of the DFE and EE point in terms of R_0. In Section 5, we perform a sensitivity study. We presented a numerical simulation in Section 6 that was based on estimated and assumed parameters. Finally, we delivered our manuscript's conclusion or discussion in section 7.

Table 1: Description of variables in the model

Variables	Interpretation
S	Susceptible population
I	The number of Infectious human decrease in the population
R	Remove population
A	Asymptote Population
U	Unpublished suggestive population

Table 2: Description of parameters in the model

Parameters	Interpretation
$bN = \phi \times N(0)$	The net inflow of susceptible populations
β_s	Probable disease transmission rate
δ_a	The modified factor for asymptote populations
δ_u	The modified factor for reported suggestive class
ϕ	The death rate of all the populations naturally
λ_a	Transition rate from asymptote to suggestive class
P_i	Fraction rate of asymptote infectious to become reported suggestive infectious
η_i	Time reported for suggestive populations have symptom
η_u	Unreported time for suggestive populations has symptoms
ξ_i	Recover reported infectious populations
ζ_u	Recover unreported infectious populations

3. MODEL FORMULATION

Infectious disease transmission dynamics are currently pervasive. Numerous academics have looked into a variety of mathematical models13–15. Now that SARS-CoV-2 has been identified, we have suggested a deterministic mathematical model of ordinary differential equations that can depict the whole impact of this new model.

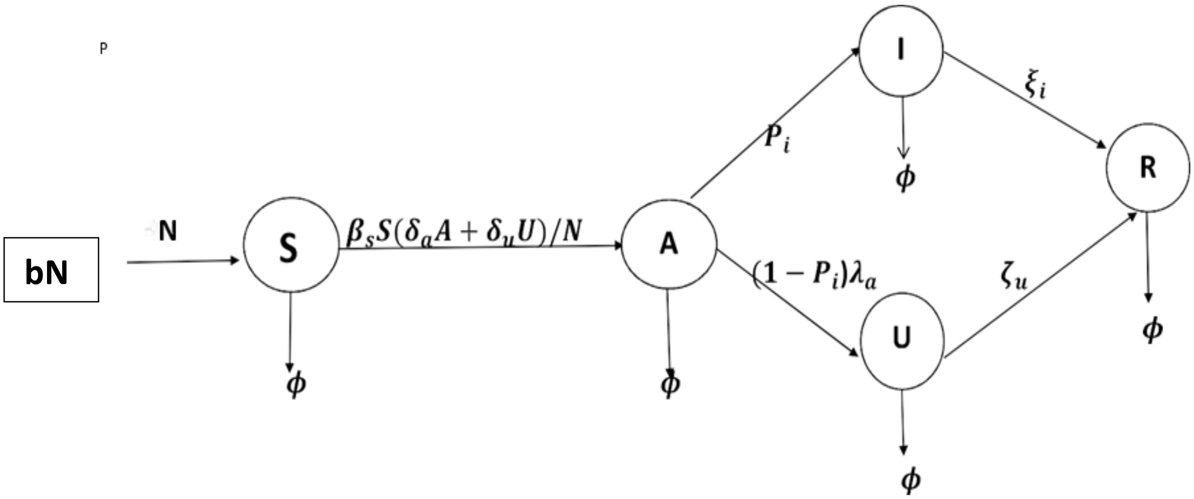

Figure 1: SAIUR Model

The uninfected or susceptible (S), asymptote (A), published suggestive infectious (I), unpublished suggestive infectious (U), and recovered persons (R) with COVID-19 disease are depicted in the schematic flow diagram (Figure 1), which implements the SAIUR model.

Here, we divided the entire human population into five groups: the susceptible population (uninfected), the asymptote population (those who do not report their infection to any health departments), the reported suggestive infected population (those who do report their infection to any health departments), the unreported suggestive infected individual (those who do not report their infection to any health departments but are ill), and the removal population (those who die from any personal causes or who have developed an immune response). SAIUR susceptible (S); asymptote (A); Infectious(I); unpublished suggestive infectious (U); and removal (R) model.
The population's total size is defined by N(t) = S(t) + A(t) + I(t) + U(t) + R(t). Consequently, the model is given in Figure 1. Nonlinear differential equations of this mode are summarized below:

$$\frac{dS(t)}{dt} = bN - \frac{\beta_s S(t)(\delta_a A(t)+\delta_u U(t)}{N(t)} - \phi S(t),$$
$$\frac{dA(t)}{dt} = \frac{\beta_s S(t)(\delta_a A(t)+\delta_u U(t)}{N(t)} - \lambda_a A(t) - \phi A(t),$$
$$\frac{dI(t)}{dt} = P_i \lambda_a A(t) - \eta_i I(t) - \phi I(t) - \xi_i I(t),$$
$$\frac{dU(t)}{dt} = \lambda_a (1 - P_i) A(t) - \eta_u U(t) - \zeta_u U(t) - \phi U(t)$$

and

$$\frac{dR(t)}{dt} = \zeta_u U(t) + \xi_i I(t) - \phi R(t) \quad (1)$$

Combining all the equations, we get

$$\frac{dN(t)}{dt} = bN - \phi N(t) - \eta_i I(t) - \eta_u U(t) \quad (2)$$

In the primary circumstances

$$S(t_0) = S_0, A(t_0) = A_0, I(t_0) = I_0, U(t_0) = U_0, R(t_0) = R_0 \quad (3)$$

Here, we take the model system's initial conditions of eq. (1). Numerous metrics change throughout time as a result of the control procedures. By taking into account a natural decay rate is less than 0 in each of the five people, we were able to calculate the statistical effects in our model. Moreover, the above model includes a net influx of the risky population, which is predicted to include new babies at a rate of bN/t. Otherwise, more interaction between uninfected populations can reduce infection in the uninfected community. In the model, and are the respective infection transference coefficients. Now, using two correction factors for the asymptote group and the infectious population with unreported symptoms, we simultaneously analyzed the transmission coefficient (which carried both the infection and contact rates of the virus). The asymptote infectious population evolved into a reported suggestive infectious population rate (P_i)with a fraction and unreported suggestive infectious population rate, respectively. The fraction rate is greater than zero and less than one. The effect of parameter and are clearly expressed of the government restrictions (like social distance, use of mask, lockdown, shaking hands, etc.) are helpful to avoid some virus. Therefore, the average amount of time spent by asymptote infectious people is days. The unreported suggestive infected populations (U) have a weekly basis, whereas the reported suggestive infectious populations (I) have a weekly basis. The clearance rates for the infected populations with reported symptoms and those without reported symptoms are, respectively.

4. EQUILIBIRIUM ANALYSIS

For equilibrium analysis, let us equate all the equations of (1) and (2) are equal to zero. We denote the equilibrium population of Susceptible S(t), Asymptote (little suggestive but not to be clinically detected) A(t), Suggestive individuals I(t), Unreported Suggestive individuals U(t), Recovered or Removal population R(t), Total population N(t).

$$bN - \frac{\beta_s S(t)(\delta_a A(t)+\delta_u u(t)}{N(t)} - \phi S(t) = 0,$$
$$\frac{\beta_s S(t)(\delta_a A(t)+\delta_u u(t)}{N(t)} - \lambda_a A(t) - \phi A(t) = 0,$$
$$P_i \lambda_a A(t) - \eta_i I(t) - \phi I(t) - \xi_i I(t) = 0,$$
$$(1-P_i)\lambda_a A(t) - \eta_u U(t) - \zeta_u U(t) - \phi U(t) = 0,$$
$$\zeta_u U(t) + \xi_i I(t) - \phi R(t) = 0$$

and

$$bN - \phi N(t) - \eta_i I(t) - \eta_u U(t) = 0 \quad (4)$$

the model given by (3) has two possibilities for the values of S, A, I, U, R and N respectively which are given below:

4.1 Population is dies out

$S(t)= I(t)= A(t)= R(t)= U(t)= N(t)=0$

4.2 When disease is prevalent

$S(t) = bN\phi^{-1}(b),$ $\quad A(t), I(t), U(t) = 0$

$R = 0,$

So

$S(t)$=N(t)

4.3When Asymptote is not present

$$bN - \frac{\beta_s S(t)(0+\delta_u U(t)}{N(t)} - \phi S(t) = 0,$$
$$S(t) = \frac{bN.N(t)}{\beta_s \delta_u U(t)+\phi},$$
$$I(t) = 0, U(t) = 0, R(t) = 0$$

4.4 When the reported infectious population is not present

(i) $S(t) = \frac{bN.N(t)}{\beta_s(\delta_a A(t)+\delta_u U(t)+\phi},$

(ii) $A(t) = \frac{\beta_s S(t)\delta_u U(t)}{N(t)(\lambda_a+\phi)-\beta_s\delta_a S(t)},$

(iii) $U(t) = \frac{\lambda_a(1-P_i)(\beta_s S(t)\delta_u U(t))}{(\eta_u+\phi+\zeta_u)(N(t)(\lambda_a+\phi)-\beta_s\delta_a S(t))},$

(iv) $R(t) = \frac{\zeta_u U(t)}{\phi}.$

4.5 When unreported infectious population is not present

(i) $S(t) = \frac{bN.N(t)}{\beta_s\delta_a A(t)+\phi},$

If A(t) =0 $\quad N(t) = \frac{bN}{\phi},$

$S(t) = \frac{b^2 N^2}{\phi^2}$,

(ii) $A(t)\left[\frac{\beta_s S(t)\delta_a}{N(t)} - \lambda_a - \phi\right] = 0$,

We get,

A(t) =0 and

$S(t) = \frac{(\lambda_a+\phi)N(t)}{\beta_s \delta_a}$,

(iii) $I(t) = \frac{P_i \lambda_a A(t)}{\eta_i+\phi+\xi_i}$,

When A(t) =0

I (t) = 0

(iv) $R(t) = \frac{\xi_i I(t)}{\phi}$,

When I(t) = 0,

So, R(t) = 0.

4.6 When Asymptote and Unreported is not present

$S(t) = \frac{bN}{\phi}$,

$I(t) = 0$,

$R(t) = 0$

4.7 When the disease is present and maintain the volume of the population

(i) $bN - \frac{\beta_s S(t)\left(\delta_a A(t)+\delta_u U(t)\right)}{N(t)} - \varphi S(t) = 0$,

(ii) $\frac{\beta_s S(t)\left(\delta_a A(t)+\delta_u U(t)\right)}{N(t)} - \lambda_a A(t) - \phi A(t) = 0$,

(iii) $P_i \lambda_a A(t) - \eta_i I(t) - \phi I(t) - \xi_i I(t) = 0$,

(iv) $(1-P_i)\lambda_a A(t) - \eta_u U(t) - \phi U(t) - \zeta_u U(t) = 0$

(v) $\xi_i I(t) + \zeta_u U(t) - \phi R(t) = 0$

Where we take $\frac{\beta_s S(t)(\delta_a A(t)+\delta_u u(t)}{N(t)} = M$

So, we get $S(t) = \frac{bN}{M+\phi}$,

$A(t) = \frac{M}{\lambda_a+\phi}$,

$I(t) = \frac{P_i \lambda_a M}{(\lambda_a+\phi)(\eta_i+\phi+\xi_i)}$,

$U(t) = \frac{\lambda_a(1-P_i)M}{(\lambda_a+\phi)(\eta_u+\phi+\zeta_u)}$,

$R(t) = \left(\frac{\lambda_a M}{\lambda_a+\phi}\right)\left(\frac{\xi_i P_i}{\eta_i+\phi+\xi_i} + \frac{\zeta_u(1-P_i)}{\eta_u+\phi+\zeta_u}\right)$.

Analyses sustain the population's size.$(S(t), I(t), A(t), R(t), u(t) > 0)$.

5. QUALITATIVE PROPERTIES OF THE MODEL

We will now investigate all of the system's variables that are non-negative for every time t with beginning circumstances. (S (0), A (0), I (0), U (0), R (0))$\in R_+^5$.

Theorem 5.1: All the solution of the system S(t), A(t), I(t), U(t), R(t) with using the primary values (3) satisfied $S(t), A(t), I(t), U(t), R(t) > 0$ for all $t > 0$.

Proof: Then the first equation of the system becomes

$$\frac{\boldsymbol{dS(t)}}{\boldsymbol{d(t)}} = \boldsymbol{bN} - \frac{\boldsymbol{\beta_s S(t)(\delta_a A(t)+\delta_u U(t)}}{\boldsymbol{N(t)}} - \boldsymbol{\phi S(t)},$$

$$= \boldsymbol{bN} - \boldsymbol{\alpha_1(t)S(t)},$$

Where

we obtained after integration,

(a) $S(t) = S_0 \exp(-\int_0^t \alpha_1(s)ds) + bN \exp(-\int_0^t \alpha_1(s)ds) \int_0^t e^{\int_0^s \alpha_1(u)du} ds > 0$

It showed that S(t) is non-negative for each t.

(b) $\frac{dA(t)}{dt} = \frac{\beta_s S(t)}{N(t)}(\delta_a A(t) + \delta_u U(t)) - \lambda_a A(t) - \phi A(t),$

$\frac{dA(t)}{dt} \geq -(\lambda_a + \phi)A(t),$

which gives

$A(t) = A_0 \exp(-\int_0^t (\lambda_a + \phi)ds) > 0$

(c) As the third equation of the system

$\frac{dI(t)}{dt} = P_i \lambda_a A(t) - \eta_i I(t) - \phi I(t) - \xi_i I(t),$

$\frac{dI(t)}{dt} \geq -(\eta_i + \phi + \xi_i)I(t),$

this inequality suggested that

$I(t) = I_0 \exp(-\int_0^t (\eta_i + \phi + \xi_i)ds) > 0$

(d) Similarly, from the fourth equation of the model

$\frac{dU(t)}{dt} = \lambda_a(1 - P_i)A(t) - \eta_u U(t) - \phi U(t) - \zeta_u U(t),$

$\frac{dU(t)}{dt} \geq -(\eta_u + \phi + \zeta_u)U(t),$

and this inequality suggested that,

$U(t) = U_0 \exp(-\int_0^t (\eta_u + \phi + \zeta_u)ds) > 0,$

(e) Last equation of our model system

$\frac{dR(t)}{dt} = \xi_i I(t) + \zeta_u U(t) - \phi R(t),$

$\frac{dR(t)}{dt} \geq -(\phi)R(t),$

and this inequality suggest that

$R(t) = R_0 \exp(-\int_0^t (\phi)ds) > 0$

From the above evaluation, we have find the solution of the system and it continue to be zeal for all $t > 0$.

5.1 Boundedness

We will satisfy with the solutions of given below theorem in the system (1) is bounded with non-negative primary values.

Theorem 5.2 The solution of the system (1) with the basic phenomena (2) which implies with R_+^5 in consistently bounded with zeal sense γ.

Proof. Now, we will investigate all the useful solution which are consistently bounded in γ to the alacrity of solutions, we get

$\frac{dS(t)}{dt} \leq bN - \phi S(t),$

It implies that

$\lim_{t\to\infty} Sup S(t) \leq \frac{bN}{\phi}$

taking $\phi = \min\{\phi.\eta_i + \phi.\eta_u + \phi.\xi_i + \phi.\zeta_u + \phi\}$

We obtain

$$\frac{dN(t)}{dt} \leq bN - \phi N(t),$$

Which gives

$$\lim_{t\to\infty} Sup N(t) \leq \frac{bN}{\phi}$$

In this solution we get a useful bounded solution in this region,

$$\gamma = \{S(t) = I(t) = U(t) = A(t) = R(t) \in R^5_{+,}(S(t) + I(t) + A(t) + U(t) + R(t)) \leq \frac{bN}{\phi}\} \quad (5)$$

Therefore, all the solution of the system is in R_+^5 will γ with in limited time. In this around of γ, gives the result of the existence, with singularity and consistently and it holds the vigorous of our SAIUR model.

5.2 Basic Reproduction Number

The next generation matrix developed by Vanden Driessche and Watmough can be used to control the fundamental R0 for the SAIUR model (2002). In this context, we look at the non-negative matrix and the non-singular M-matrix as representing, respectively, the formation of infection with the transition portion. We refer to this system (1) as the SAIUR model system.

$$F = \begin{bmatrix} \frac{\beta_s S(T)(\delta_a A(t) + \delta_u U(t))}{N(t)} \\ 0 \\ 0 \end{bmatrix},$$

and

$$V = \begin{bmatrix} (\lambda_a + \phi)A(t) \\ -P_i\lambda_a A(t) + (\eta_i + \phi + \xi_i)I(t) \\ -(1-P_i)\lambda_a A(t) + (\eta_u + \phi + \zeta_u)U(t) \end{bmatrix},$$

Now F and V can be expressed as

$$F = \begin{pmatrix} \beta_s\alpha_a & 0 & \beta_s\alpha_u \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and

$$V = \begin{pmatrix} \lambda_a + \phi & 0 & 0 \\ -P_i\lambda_a & \eta_i + \phi + \xi_i & 0 \\ -(1-P_i)\lambda_a & 0 & \eta_u + \phi + \zeta_u \end{pmatrix},$$

Diekmann et al., 2000 define R_0 is the spectral radius of the next generation matrix:

$$R_0 = \rho(FV^{-1})$$

$$= \begin{pmatrix} \beta_s\alpha_a & 0 & \beta_s\alpha_u \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\lambda_a+\phi} & 0 & 0 \\ \frac{P_i\lambda_a}{(\lambda_a+\phi)(\eta_i+\phi+\xi_i)} & \frac{1}{(\eta_i+\phi+\xi_i)} & 0 \\ \frac{(1-P)_i\lambda_a}{(\lambda_a+\phi)(\eta_u+\phi+\zeta_u)} & 0 & \frac{1}{(\eta_u+\phi+\zeta_u)} \end{pmatrix},$$

$$R_0 = \frac{\beta_s\alpha_u}{\eta_u + \phi + \zeta_u} + \frac{(\beta_s\alpha_a)(\eta_u + \phi + \zeta_u) + \beta_s\alpha_u(1-P_i)\lambda_a}{(\lambda_a + \phi)(\eta_u + \phi + \zeta_u)}.$$

5.3 Analysis of stability of DFE

Theorem5.3. When $R_0 > 1$ for the DFE$E_0(bN/\phi, 0,0,0,0)$are exists and it is locally asymptote stable, else it is unstable.

Proof. We calculate the Jacobian Matrix of the system to governed the local stability of $E_0(bN/\phi, 0,0,0,0)$ for the DFE is given by

$$J_{E_0} = \begin{pmatrix} -\phi & -\beta_s\delta_a & 0 & -\beta_s\delta_u & 0 \\ 0 & \beta_s\delta_a - (\lambda_a + \phi) & 0 & \beta_s\delta_u & 0 \\ 0 & P_i\lambda_a & -(\eta_i + \phi + \xi_i) & 0 & 0 \\ 0 & \lambda_a(1 - P_i) & -(\eta_u + \phi + \zeta_u) & 0 & 0 \\ 0 & 0 & \xi_i & \delta_u & -\phi \end{pmatrix},$$

The characteristics equation in the corresponding of J_{E_0} for the eigen value μ is $\det(J_{E_0} - \mu I_5) = 0$

5.4 Analysis of Global Stability for DFE

In this part, we calculate the global stability for the DFE point E_0 with the condition is $R_0 < 1$. In this we use a Lypanov function likely to be used by Mcclusky (2010) khajanchi & Banerjee (2017) and korobeinikov& Maini (2004) Lyapunov function take superiority of all the stuff of the functions**.**

$$r(k) = k - 1 - \ln(k)$$

which is non-negative in R_+^5 until at k=1, where it becomes zero. The following theorem proves global equilibrium stability.

Theorem 5.4 The DFE$\in_0$ of the SAIUR system (1) is globally asymptote stable if $R_0 < 1$ and$\beta_s\delta_u < \eta_i + \phi < \beta_s\delta_u(1 - P_i) < \phi$.

Proof. Examine the Lyapunov function

$$V_{E_0}(S(t) = A(t) = I(t) = U(t) = R(t)) = S_0 r\left(\frac{S(t)}{S_0}\right) + A(t) + I(t) + U(t) + R(t)$$

where V_{E_0} is always non-negative in the around γ and attains 0 at E_0. Now we differentiate V_{E_0} through the solution trajectory to that V_{E_0} is negative definite

$$V_{E_0} = \left(1 - \frac{S_0}{S_t}\right)S^*(t) + A^*(t) + I^*(t) + U^*(t) + R^*(t),$$

$$= \left(1 - \frac{S_0}{S(t)}\right)(bN - \frac{\beta_s S(t)}{N(t)}(\delta_a A(t) + \delta_u U(t)) - \phi S(t) + \frac{\beta_s S(t)}{N(t)}(\delta_a A(t) + \delta_u U(t))$$

$$-\lambda_a A(t) - \phi A(t) + P_i\lambda_a A(t) - \eta_i I(t) - \phi I(t) - \xi_i I(t) + \lambda_a A(t) - P_i\lambda_a A(t) - \eta_u U(t),$$

$$-\phi U(t) - \zeta_u U(t) + \xi_i I(t) + \zeta_u U(t) - \phi R(t))$$

$$V_{E_0} = \left(1 - \frac{S_0}{S(t)}\right)(bN - \phi S(t) - \phi A(t) - \eta_i I(t) - \phi I(t) - \eta_u U(t) - \phi U(t) - \phi R(t)),$$

$$V_{E_0} = bN - \phi S(t) - \phi A(t) - (\eta_i + \phi)I(t) - (\eta_u + \phi)U(t) - \phi R(t),$$

$$V_{E_0} = bN - \phi S(t) - \phi A(t) - (\eta_i + \phi)I(t) - (\eta_u + \phi)U(t) - \phi R(t),$$

$$= \frac{-bNS_0}{S(t)} + \phi S_0 + \frac{\phi S_0}{S(t)}A(t) + \frac{\eta_i S_0 I(t)}{S(t)} + \frac{\phi S_0 I(t)}{S(t)} + \frac{\eta_u S_0 U(t)}{S(t)} + \frac{\phi S_0 U(t)}{S(t)} + \frac{\phi S_0 R(t)}{S(t)},$$

$V_{E_0} > 1$ if (i) $R_0 < 1$and$\beta_s\delta_u < \eta_i + \phi < \beta_s\delta_u(1 - P_i) < \phi$,

With the aid of the relation with A.M and G.M, we make certain that $V_{E_0} \leq 0$ and equality holds at E_0. Hence $R_0 < 1$ when DFE is globally asymptote stable.

5.5 Perseverance of the Covid-19

In the theorem 4.3 we verified that basic $R_0 < 1$, at the initial point of epidemic the virus is dies out, if $R_0 > 1$ then the disease will be spread more and more. Likely to be assumed that A(t), I(t), U(t) will always remain persistent to this event. Now we will show the following theorem to determine the perseverance of the disease.

Theorem5.5. Let $R_0 > 1$, the Corona virus disease will be uniformly perseverance in the sense that there exists an $K > 0$ in every positive solution of the system (1) $\liminf_{t\to\infty} A(t) > 0, \liminf_{t\to\infty} I(t) > 0$ and $\liminf_{t\to\infty} U(t) > 0$.

Proof. With the help of perseverance theorem[20] we prove uniform perseverance, in the basis we consider that

$$K = (S(t), A(t), I(t), U(T), R(t))$$

$$\overline{K} = (A(t), I(t), U(T))$$

$L = \{K_j \in R_+^5 | K_j \geq 0|, j = 1 \ldots 5$ where O_j is the jth component of O$\}$
$L_0 = \{K \in L | K_j > 0|, j = 2,3,4\}$,
$M = L/L_0 = \{K \in L | K_j = 0|$, for some $j = 2,3,4\}$.

Now we want to demonstrate the system (1) is uniformly perseverance with regard to (L_0, M). Since M has a unique equilibrium. It is enough to determine that $W^S(E_0) \bigcap L_0 = \gamma$ where $W^S(E_0)$ signify the stable manifold of DFE E_0.

Let it is not true, then find the solution for $(S(t), I(t), A(t), U(t), R(t)) \in L_0$ of the model (1) such that

$$\lim_{t\to\infty}\left((S(t), I(t), A(t), U(t), R(t))\right) \to (bN/\phi, 0,0,0,0),$$

Then for any $\in > 0$, we determine

$$\frac{bN}{\phi} - \in \leq S(t) \leq \frac{bN}{\phi} + \in,$$
$$0 \leq K_j \leq \in, j = 2,3,4$$

For several large values of t in the model (1), the following holds:

$$\begin{pmatrix} dA(t)/dt \\ dI(t)/dt \\ dU(t)/dt \end{pmatrix} = \begin{pmatrix} \beta_s \frac{S(t)(\delta_a A(t) + \delta_u U(t))}{N(t)} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -(\lambda_a + \phi) & 0 & 0 \\ P_i \lambda_a & -(\eta_i + \phi + \xi_i) & 0 \\ \lambda_a(1 - P_i) & -(\eta_u + \phi + \zeta_u) & -(\eta_u + \phi + \zeta_u) \end{pmatrix} \begin{matrix} A(t) \\ I(t) \\ U(t) \end{matrix}$$
$$\geq \begin{pmatrix} \beta_s \delta_a \tilde{S}(\in) - (\lambda_a + \phi) & 0 & \beta_s \delta_u S^{\sim}(\in) \\ P_i \lambda_a & -(\eta_i + \phi + \xi_i) & 0 \\ \lambda_a(1 - P_i) & 0 & -(\eta_u + \phi + \zeta_u) \end{pmatrix}$$

where

$$\tilde{S}(\in) = \frac{bN/\phi - \in}{bN/\phi + \in}$$

and

$$\tilde{J}(0) = \begin{pmatrix} \beta_s \delta_a - (\lambda_a + \phi) & 0 & \beta_s \delta_u \\ P_i \lambda_a & -(\eta_i + \phi + \xi_i) & 0 \\ \lambda_a(1 - P_i) & 0 & -(\eta_u + \phi + \zeta_u) \end{pmatrix},$$

It is necessary to note down that , $\tilde{J}(0)$is equal to (F-V) has at least one eigen value with positive real part when $R_0 > 1$ .Therefore $\in > 0$ is arbitrary, and it can make $\in$ small enough so that $\chi\tilde{J}(\in)$ is positive where $\chi(B)$ is the greatest real part of the eigen value of B. So there exist solutions of the linear system

$$\frac{d\bar{K}}{dt} = \tilde{J}(\in)\bar{K}$$

It can grow aggressively. In the comparison the solution, **K** becomes unbounded as $t \to \infty$. It contradicts our assumption that the model (1)'s solutions are uniformly constrained. Hence $W^S(E_0) \cap L_0 = \gamma$ with the help of the theorem (4.6) in[20] and it can be inferred the system (1) is uniformly perseverance with regards to(L_0, M). Therefore, the system of eq. (1) is lavish (showed in the theorem 2) and thus by using the theorem 3.3 in[21] indicates the model of the system (1) for an interior equilibrium point and it showed that all components are positive and it is absolute the proof of the theorem.

5.6 Existence of an EE point

The SAIUR model system (1) has an EE point $E^*(S^*, A^*, I^*, U^*, R^*)$ with the positive components and says that the $R_0 > 1$ then equating all the derivatives of the model system (1) to zero and solving the equations, then we get where we take $\frac{\beta_s S(t)(\delta_a A(t) + \delta_u u(t))}{N(t)} = \text{M}$, (6)

so we get

$$S(t) = \frac{bN}{M + \phi},$$
$$A(t) = \frac{M}{\lambda_a + \phi},$$
$$I(t) = \frac{P_i \lambda_a M}{(\lambda_a + \phi)(\eta_i + \phi + \xi_i)},$$
$$U(t) = \frac{\lambda_a(1 - P_i)M}{(\lambda_a + \phi)(\eta_u + \phi + \zeta_u)},$$

$$R(t) = \left(\frac{\lambda_a M}{\lambda_a+\phi}\right)\left(\frac{\xi_i P_i}{\eta_i+\phi+\xi_i} + \frac{\zeta_u(1-P_i)}{\eta_u+\phi+\zeta_u}\right), \tag{7}$$

Taking all the expression of (7) and put in to the (6), solving all of the equations in terms of M in the following way:

$$\kappa_1 M + \kappa_2 = 0 \tag{8}$$

where,

$$\kappa_1 = (\eta_i+\phi)(\eta_u+\phi) + P_i\lambda_a(\eta_u+\phi) + (1-P_i)\lambda_a(\eta_i+\phi),$$

$$\kappa_2 = (\lambda_a+\phi)(\eta_i+\phi)(\eta_u+\phi)(1-R_0),$$

It is clearly show that the system (1) has unique positive EE point when $R_0 > 1$ and there is no equilibrium points when $R_0 < 1$.

$$\kappa_1 > 0, \lambda_a+\phi > 0, \eta_i+\phi > 0, \quad \eta_u+\phi > 0.$$

Theorem5.6. When reproduction is greater than one.

Proof

$$y_1 = S(t), y_2 = A(t), y_3 = I(t), y_4 = U(t), y_5 = R(t),$$

then the system (1) becomes

$$\frac{dy_1}{dt} = bN - \frac{\beta_s y_1(\delta_a y_3+\delta_u y_4}{N(t)} - \phi y_1 \equiv Q_1,$$
$$\frac{dy_2}{dt} = \frac{\beta_s y_1(\delta_a y_3+\delta_u y_4}{N(t)} - \lambda_a y_3 - \phi y_3 \equiv Q_2,$$
$$\frac{dy_3}{dt} = P_i\lambda_a y_3 - \eta_i y_2 - \phi y_2 - \xi_i y_2 \equiv Q_3,$$
$$\frac{dy_4}{dt} = \lambda_a(1-P_i)y_3 - \eta_u y_4 - \phi y_4 - \zeta_u y_4 \equiv Q_4$$

and

$$\frac{dy_5}{dt} = \xi_i y_2 + \zeta_u y_4 - \phi y_5 \equiv Q_5$$

(9)

We take $R_0 = 1$ for the bifurcation parameter the Jacobian matrix of the system (9) for the DFE E_0 at the threshold point are

$$\beta_s = \beta_s^* = \frac{(\lambda_a+\phi)(\eta_u+\phi)}{(\eta_u+\phi)\delta_a + \delta_u(1-P_i)\lambda_a}$$

So,

$$J_{E_0}^{\ *} = \begin{pmatrix} -\phi & -\beta_s^*\delta_a & 0 & -\beta_s^*\delta_u & 0 \\ 0 & \beta_s^*\delta_a - (\lambda_a+\phi) & 0 & \beta_s^*\delta_u & 0 \\ 0 & P_i\lambda_a & -(\eta_i+\phi+\xi_i) & 0 & 0 \\ 0 & \lambda_a(1-P_i) & -(\eta_u+\phi+\zeta_u) & 0 & 0 \\ 0 & 0 & \xi_i & \delta_u & -\phi \end{pmatrix},$$

The eigenvalues of the $J_{E_0}^*$ are $-\phi, -(\eta_u+\phi), (2\phi+\lambda_a-\beta_s^*\delta_a)$ negative real parts and 0 is the simple eigen value. So the centre manifold theorem applied to get a right eigen vectors and left eigen vector corresponds to the left eigen vector is given by

$$v = \left[-R_0\left(1+\frac{\lambda_a}{\phi}\right) \quad 1 \quad \frac{P_i\lambda_a}{\eta_i+\phi} \quad \frac{(1-P_i)\lambda_a}{\eta_u+\phi}\right]T_{v_2},$$

and $$u = \left[0 \quad 1 \quad 0 \quad \frac{\beta_s^*\delta_u}{\eta_u+\phi}\right]u_2 S,$$

Hence,

$$a = \sum_{k,i,j=1}^{5} u_k v_i v_j \left[\frac{\partial^2 q_k}{\partial x_i \partial x_j}(E_0)\right] \quad \text{and} \quad b = \sum_{k,i=1}^{5} u_k v_i \left[\frac{\partial^2 q_k}{\partial x_i \partial \beta_s^*}(E_0)\right],$$

the sign of local stability criteria of the EE point E^* and replacing all the values of second order to measured DFE, E_0 is define by

$$a = 2\left[v_1 v_2 \frac{\partial^2 q_2}{\partial x_1 \partial x_2} + v_1 v_4 \frac{\partial^2 q_2}{\partial x_1 \partial x_4}\right] u_2,$$
$$= -2R_0\left(1 + \frac{\lambda_a}{\phi}\right)\left[\beta_s^* \delta_a + \frac{\lambda_a (1 - P_i)}{\eta_u + \phi}\right] u_2 v_2 < 0,$$

and

$$b = \left[v_2 \frac{\partial^2 q_2}{\partial x_2 \partial \beta_s^*} + v_4 \frac{\partial^2 q_2}{\partial x_4 \partial \beta_s^*}\right] u_2,$$
$$= \frac{R_0\left(\lambda_a + \phi\right)}{\beta_s^*} x_1 u_2 v_2 > 0.$$

Thus, $b > 0$ and $a > 0$ at $\beta_s = \beta_s^*$. So, a transcritical bifurcation is occur at $(R_0 = 1)$ and unique EE is locally asymptote established for $(R_0 > 1)$.

5.7 Worldwide stability of EE point

The worldwide stability analysis of an EE point E^* when the $(R_0 > 1)$. In this way we use a Lyapunov functional to define the properties of the function. Now we will proof the following theorem:

Theorem5.7. If $R_0 > 1$ then the equation (1) of the system has EE point E^*.

Proof. Let E^* exist for the well-defined system in R_+^5, so we demonstrate a Lyapunov function[22] as

$$V_{E^*}\left(S, A, I, U, R\right) = S^* r\left(\frac{S}{S^*}\right) + A^* r\left(\frac{A}{A^*}\right) + I^* r\left(\frac{I}{I^*}\right) + U^* r\left(\frac{U}{U^*}\right) + R^* r\left(\frac{R}{R^*}\right)$$

Now differentiate V_{E^*} with the solution trajectories of the model we get,

$$V_{E^*} = \left(1 - \frac{S}{S^*}\right) S^* + \left(1 - \frac{A}{A^*}\right) A^* + \left(1 - \frac{I}{I^*}\right) I^* + \left(1 - \frac{U}{U^*}\right) U^* + \left(1 - \frac{R}{R^*}\right) R^*$$
$$= \left(1 - \frac{S}{S^*}\right)\left(bN - \frac{\beta_s S}{N}(\delta_a A + \delta_u U) - \phi S\right) + \left(1 - \frac{A}{A^*}\right) + \left(1 - \frac{I}{I^*}\right)(P_i \lambda_a A - \eta_i I - \phi I - \xi_i I) + \left(1 - \frac{U}{U^*}\right)(\lambda_a (1 - P_i) A - \eta_u U - \phi U - \zeta_u U) + \left(1 - \frac{R}{R^*}\right)(\xi_i I + \zeta_u U - \phi R)$$

On solving with the relation of AM and GM we get the equality holds at E_* is globally asymptote stable and we make sure that $V_{E^*} \leq 0$.

6. SENSITIVE STUDY

We introduced the nature of the model to investigate the R_0 and it is essential to understand the important factors about the transmission of disease. Now we perform a sensitivity study of the SAIUR model with parameters to identify the impact on the basic R_0 and on the disease transmission. The normalized sensitivity index of R_0 with a parameter δ_u as defined below:

$$\prod_{\delta_u}^{R_0} = \frac{\partial R_0}{\partial \delta_u} \times \frac{\delta_u}{R_0}$$

Similarly for the normalized sensitivity index of R_0 with a parameter δ_a in the SAIUR system of the model. Now we analyse for the β_s by a given percentage increasing (decreasing) always R_0 by the same percentage.

$$\prod_{\beta_s}^{R_0} = \frac{\partial R_0}{\partial \beta_s} \times \frac{\beta_s}{R_0} = 1$$

The sensitivity analysis of the effective parameters for relevant quantitative changes. In the other side, the estimation of the parameter is beside small for the sensitivity indices. With this analysis we get some sensitive parameter of the SAIUR model are β_s, ϕ, δ_a and δ_u. When we increase the value of δ_u or δ_a, it will increase the basic R_0 and if we increase the value of ϕ, it will decrease the basic R_0.

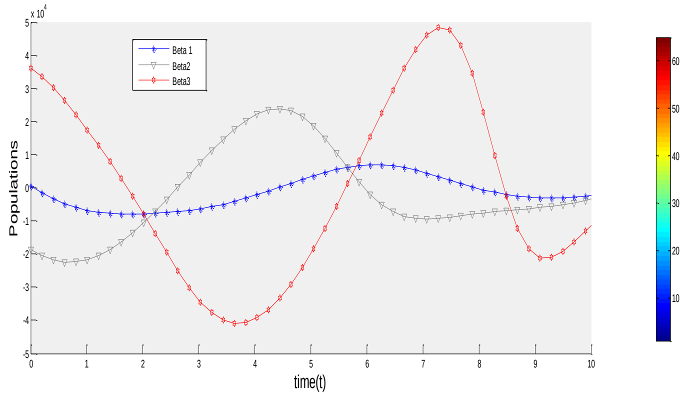

Figure 2 : S,A,I,U and R at the time with the basic conditions S(0)=18320,A(0)=1000,I(0)= 150,U(0)=30,R(0)=200and the parameters: bN=2500**,**

$\beta_1 = 5, \beta_2 = 4.25, \beta_3 = 3.86, \delta_a = 0.4775, \delta_u = 0.695, \phi = 0.62, \lambda_a = 0.29, \eta_i = 111.11,$
$\xi_i = 0.06, \zeta_u = 0.04, \eta_u = 20, P_i = 0.078$

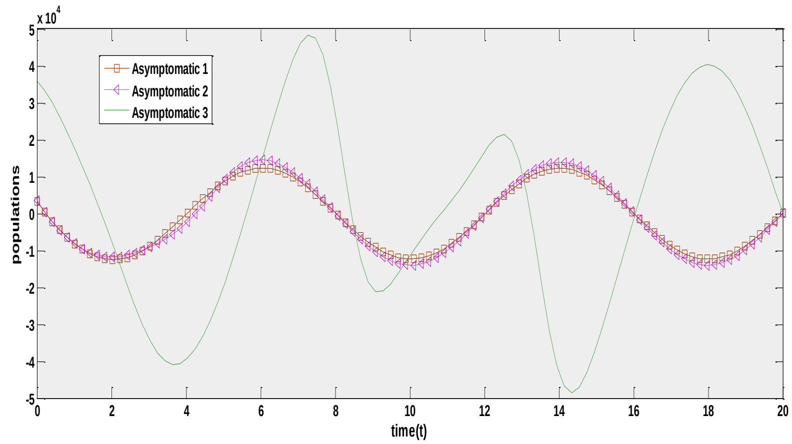

Figure 3: S,A,I,U and R at time with the basic conditions S(0)=18320,A(0)=1000,I(0)= 150,U(0)=30,R(0)=200and the parameters: bN=2500,

$\beta_s = 3.86, \delta_{a1} = 0.4775, \delta_{a2} = 2.3475, \delta_{a3} = 1.775, \delta_u = 0.695, \phi = 0.62, \lambda_a = 0.29, \eta_i = 111.11,$
$\xi_i = 0.06, \zeta_u = 0.04, \eta_u = 20, P_i = 0.078$

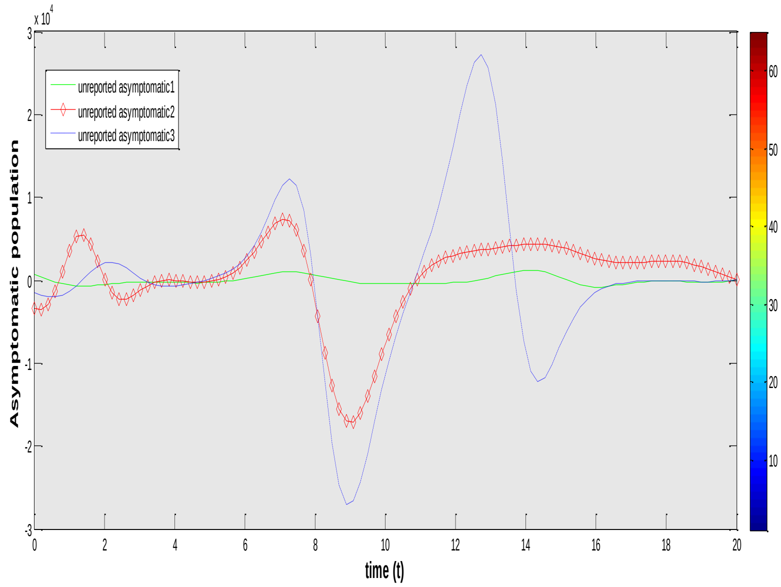

Figure 4: S,A,I,U and R at time with the basic conditions S(0)=18320,A(0)=1000,I(0)= 150,U(0)=30,R(0)=200and the parameters: bN=2500,

$$\beta_s = 3.86 \delta_s = 0.4775, \delta_{u1} = 0.695, \delta_{u2} = 1.125, \delta_{u3} = 2.480, \ \phi = 0.62, \lambda_a = 0.29, \eta_i = 111.11,$$
$$\xi_i = 0.06, \zeta_u = 0.04, \eta_u = 20, P_i = 0.078$$

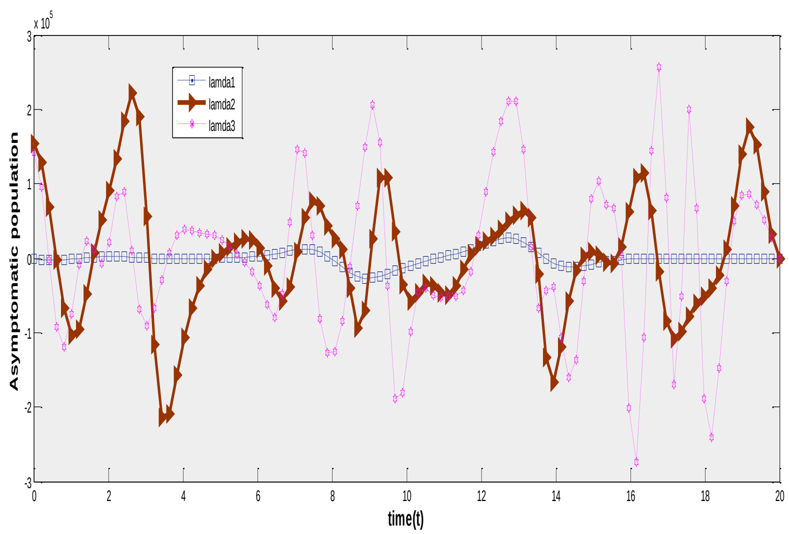

Figure 5: S,A,I,U and R at time with the basic conditions S(0)=18320,A(0)=1000,I(0)= 150,U(0)=30,R(0)=200and the parameters: bN=2500**,**

$$\beta_s = 3.86 \delta_s = 0.4775, \ \phi = 0.62, \lambda_{a1} = 0.29, \lambda_{a2} = 2.25, \lambda_{a3} = 4.86, \eta_i = 111.11,$$
$$\xi_i = 0.06, \zeta_u = 0.04, \eta_u = 20, P_i = 0.078$$

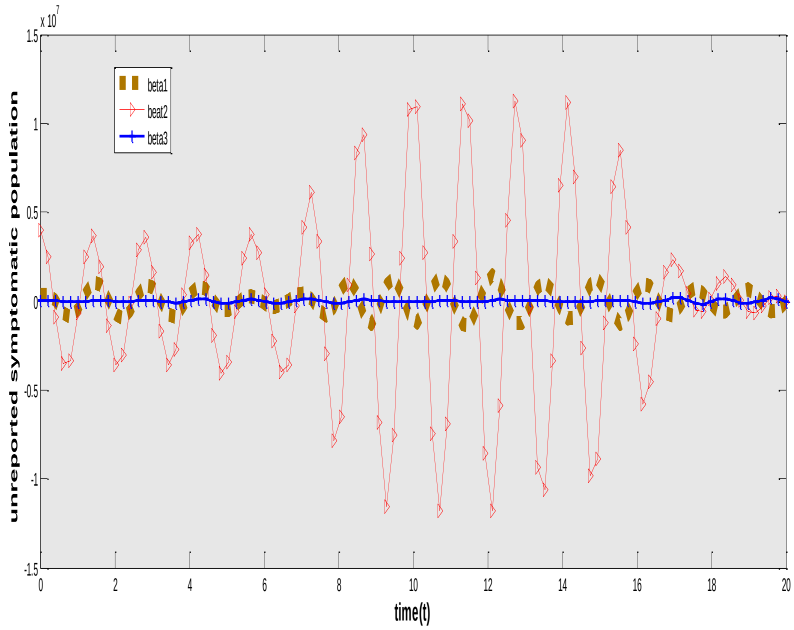

Figure 6: S,A,I,U and R at time with the basic conditions S(0)=18320,A(0)=1000,I(0)= 150,U(0)=30,R(0)=200and the parameters: bN=2500**,**

$\phi = 0.62, \lambda_a = 0.29, \eta_i = 111.11$ $\phi = 0.62, \lambda_a = 0.29, \eta_i = 111.11,$
$\xi_i = 0.06, \zeta_u = 0.04, \eta_u = 20, P_i = 0.078$

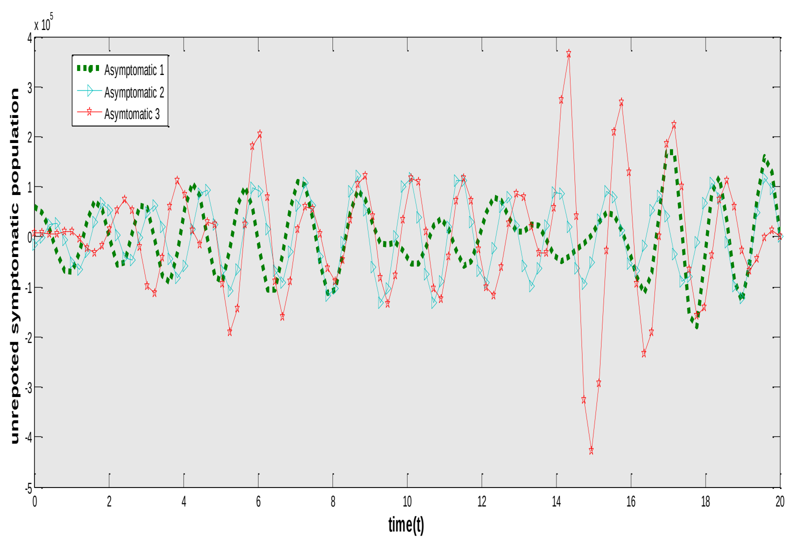

Figure 7: S,A,I,U and R at time with the basic conditions S(0)=18320,A(0)=1000,I(0)= 150,U(0)=30,R(0)=200and the parameters: bN=2500,

$\beta_s = 3.86, \delta_{a1} = 0.4775, \delta_{a2} = 2.3475, \delta_{a3} = 1.775, \delta_u = 0.695,$ $\beta_1 = 5, \beta_2 = 4.25, \beta_3 = 3.86, \delta_a = 0.4775, \delta_u = 0.695,$,
$\xi_i = 0.06, \zeta_u = 0.04, \eta_u = 20, P_i = 0.078$

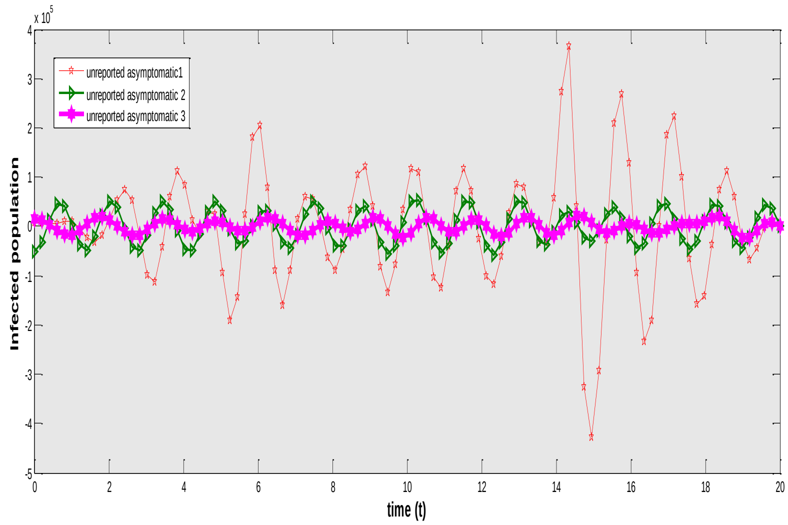

Figure 8 : S,A,I,U and R at time with the basic conditions S(0)=18320,A(0)=1000,I(0)= 150,U(0)=30,R(0)=200and the parameters: bN=2500,

$\beta_s = 3.86 \delta_s = 0.4775, \delta_{u1} = 0.695, \delta_{u2} = 1.125, \delta_{u3} = 2.480,$ $\beta_1 = 5, \beta_2 = 4.25, \beta_3 = 3.86, \delta_a = 0.4775, \delta_u = 0.695,$,
$\xi_i = 0.06, \zeta_u = 0.04, \eta_u = 20, P_i = 0.078$

7. CONCLUSION

In this study, we investigated the dynamics of a SAIUR model (1) for the COVID-19, and we evaluated the value of the parameters that best match our SAIUR model using the ODE 45 test in MATLAB. The suggested SAIUR model system (1) has 12 non-negative parameter values, and we have estimated 5 of them the probability rate of disease transmission, the adjustment for asymptote individuals, the adjustment factor for reported suggestive infectious individuals, the transmission rate from asymptote to suggestive infectious individuals, the rate of reported infectious with symptoms, the rate of unreported infectious with symptoms, and the F-statistic. Our analysis shows that if $R_0 < 1$ then the DFE is locally asymptotically stable. Then, our SAIUR model showed the perseverance of disease when $R_0 > 1$. For $R_0 > 1$ the EE point E^* is locally asymptote stable.

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