Global Journal of Modeling and Intelligent Computing (GJMIC)

Vol. 2, No. 2, July, 2022 ISSN: 2767-1917

OPTIMIZATION OF QUALITY OF LIFE INDEX WITH α-LEVEL OF SIGNIFICANCE OF LINEAR CONSTRAINTS

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ABSTRACT

In the present investigation, a statistical quantification approach has been presented to optimize the Quality of Life Index (QOLI) subject to some constraints in a certain region. The QOLI is based on certain parameters of vital life which measure Physical, Psychological, Economic, Education, Social, and Environmental conditions in terms of rating. Principle Component Analysis (PCA) is used for the final selection of Quality of Life (QOL) indicators. The Best Worst Method (BWM) has been employed to assign weights to different traits of QOLI and the multiple linear regression provides information about the linear constraints a with α-level of significance. The implementation of this mathematical model has been also done in the exhaustive case study of QOLI of Nagaland state of India and different values of QOLI are observed. These QOLI values may be helpful in making the policies for good quality of life for the Nagaland State and for similar places as well.

Keywords: Quality of life Index, Principal Component Analysis (PCA), Best Worst Method.

1. INTRODUCTION

The comprehension, quantification and improvement of Quality of Life (QOL) have been of paramount importance for researchers in various disciplines like economics, education psychology, sociology and environmental science. Measurement of economic performance is now being shifted from a mere income-production measurement to a meticulous QOL assessment. Policies in all countries are focused on facilitating good Quality of Life for its citizens. Several outlooks on the concepts, frameworks and measurement of QOL has been given in the literature. Economic measure, self-reported wellbeing and social wellbeing are all crucial in determining human QOL (Diener et al. 1997). The Organization for Economic Co-operation and Development (OECD) Better Life Index 2011 is one of the most recognized work which captures the multidimensional concept of QOL (Durand et al. 2015; Koronakos, 2020; Stiglitz et al., 2009).

Linear programming is a dynamic and flexible subset of mathematical programming introduced by Dantzig in 1947 and since then, it has been used for the optimization of various problems. It has several computational advantages and is suitable for solving complex systems (Luenberger et al., 2016). Linear programming is known for its simplicity. In view of its simplicity and ease to the solution, we have introduced an accessible method to formulate an α-level significance Linear Programming Problem (LPP) model. A major hurdle that stands in the way of LPP and its applicability is the linearity condition. In real-life situations, to get concrete linear relations between variables is quite impossible. Taking this into account, in this paper, we consider the linearity of the variables to an acceptable α-significance level,$\alpha\epsilon[0,1]$.

In this paper, our new contribution is in the construction of a Linear Programming Problem for QOLI optimization, given that variables are not exactly linear. The objective function of Quality of life Index (QOLI) is constructed with help of the Best Worst Method (BWM) (Rezaei, 2016). In BWM, the weights are assigned using a well-defined mathematical model, taking into consideration the preferences of the decision-makers making it a more rational method. Linear constraints are constructed using multiple regression with α-level of precision level in linearity.

A step-by-step discussion of the formulation procedure, along with a robust analysis is presented. A fresh case study for Nagaland, India is incorporated as an application of the new QOL optimization modelling technique. The whole manuscript has been presented in different sections. The assumptions and the notations necessary for the study are given in section 3. The formulation of the problem along with the selection of the QOLI and other properties are presented in section 4. The theoretical findings of the formulated problem have been verified through a real-life problem under section 6. The Algorithms required to get optimum quality of life Index are presented in Algorithms for optimum quality of life Index. The ultimate indicators used to present the problem are given in section selection of final QOL indicators. The formulation of the optimization problem for the real data is presented in the section QOLI optimization model for Nagaland. Results and analysis of the problem under consideration are discussed in section result and analysis. The final outcomes of the problem under study are presented in section conclusion and the finally paper ends with the references.

2. LITERATURE REVIEW

Slottje had brought forward a multidimensional approach of measuring QOL across countries (Slottje, 1991). Ma et al. (2020), have analyzed the difference of urban-rural QOL and have concluded that advancing and cementing rural productivity and deepening the reforms of urban-rural systems enhances the people's QOL and sense of wellbeing. There are various methods of weighting each attribute of the QOL by weighting each attribute equally, by researcher's own judgment, by using objective methods like Principal Component Analysis (PCA), by using hedonic approach with the help of regression analysis (Slottje, 1991).) An integrated PCA-DEA approach has been investigated for evaluating and ranking QOL (Põldaru et al., 2014). Tripathi et al. (2019), have set forth a parameter selection procedure for Water Quality Index (WQI) using PCA. Zhu (2001), mentioned that the multiple attributes of QOL must be weighted in a rational and objective manner while Costanza et al. Costanza et al (2007), worked on the weighting of QOL indicators and suggested that these must not only be mathematical but also should assimilate the underlying workings responsible for weight assignment. Stiglitz *et al*. suggested that, in the construction of composite indices, a crucial step is employed in assigning the weights to the indicators (Stiglitz, 2009). Rezaei (2016), has shown that a unique solution can be obtained for the case of a fully consistent comparison using a linear minimax model. Rezaei et al. (2018), introduced and worked on a novel linear BWM for assigning weights. Rezaei et al. (2018), introduced an illustrious procedure for examining comparative importance of the components of Logistics Performance Index (LPI) by using BWM to assign weights to each component of the LPI. Omrani et al. (2020), worked in the direction of forming the human development index and constructed a semi-human development index by using BWM as an operational tool to find the weights of indicators. Wang et al. (2019), proposed a two-fold methodology for constructing index, wherein the judgements on the sub-indicators are evaluated initially and then BWM is introduced.

Pun et al. (2019), introduced an easily implementable estimator for effective parameters of high dimensional portfolios with the help of linear programming, known as the Linear Programming Optimal (LPO) estimator. Zhu simplified the optimization of autonomous intersection control for a dynamic traffic assignment with connected vehicle environment by transforming the analytically difficult non-linear programming problem into a simpler linear programming problem (Zhu et al., 2015).

3. MATHEMATICAL FRAMING

3.1 Assumptions and Notations Used

i) Quality of Life is a multi-dimensional concept and requires multiple dimensions to define it.
ii) Each dimension is also multidimensional and consists of different sub-dimensions, that we call parameters.
iii) 'm' denotes the number of samples collected.
iv) $I_1, I_2, \ldots, I_r$ represents the theoretically selected QOL indicators.
v) $I_{v1}, I_{v2}, \ldots, I_{vm}$ represents the data for the v^{th} theoretically selected QOL indicator. Here $v = 1, 2, \ldots, r$.
vi) There are N number of final selected dimensions defining QOL, denoted by $X_1, X_2, \ldots, X_N$.
vii) Corresponding to each dimension X_i,, there are n_i number of components. x_{ij} represents the j^{th} component of i^{th} dimension.
viii) For each x_{ij} , we get a set of m observations represented by $x_{ij.1}, x_{ij.2}, \ldots, x_{ij.m}$.

3.2 Final Selection of QOLI Indicators

The final selection of QOL indicators is done using PCA. PCA is a technique which reduces the dimension of a given vector by projecting it to a vector of lesser dimension using orthogonal linear transformations while retaining as much information as possible. Information means the variance of the data under investigation. If $X = (X_1, X_2, \dots, X_N)$ is a vector of N random variables, PCA helps to find a corresponding vector Y=$(Y_1, Y_2, \dots, Y_M)$ such that $M < N$ and Y preserves the variance of X to an optimal level. To do this, the initial step is to find a linear transformation $U_1 : R^N \to R^M$ such that-

$$U_1(X_1, X_2, \dots, X_N) = \alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_N X_N = Y_1, \quad (1)$$

keeping in mind that the maximum variance is retained.

Equation (1) gives PC1. In the next step, we again compute Y_2 by finding the 2^{nd} linear transformation U_2 , which is not correlated with U_1 and preserves second maximum variance. We continue finding such uncorrelated linear transformations until the required m^{th} Principle Component (PC)-PCm is computed. Finally, using all the PCs, we get the reduced vector Y=$(Y_1, Y_2, \dots, Y_M)$.

Theorem: If S is the covariance matrix of a variable $X = (X_1, X_2, \dots, X_N)$, then for $k = 1,2, \dots, M$, the kth PC is given by $Y_k = {v_k}^T X$, where v_k is an eigen vector of S corresponding to its k^{th} largest eigen value λ_k and $var({v_k}^T X) = \lambda_k$.

Proof:. The above theorem can be proved by formulating the given problem as a maximization problem where the objective is to maximize covariance matrix of Y and the constraint is the orthogonality condition of U_i. After applying Lagrangian multiplier method, it finally reduces to an eigen vector problem-

$S_X U = \lambda$U, where S_X is the covariance matrix of X.

Thus, the problem of finding the kth PC reduces to finding an eigen vector of S corresponding to its k^{th} largest eigen value λ_k.

We apply PCA on the components which are theoretically selected and obtain as,

$$X_1 = C_{11}x_{11} + C_{12}x_{12} + \cdots + C_{1n_1}x_{1n_1} = \sum_{j=1}^{n_1} c_{1j}x_{1j}$$

$$X_2 = C_{21}x_{21} + C_{22}x_{22} + \cdots + C_{2n_2}x_{2n_2} = \sum_{j=1}^{n_2} c_{2j}x_{2j}$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$X_N = C_{N1}x_{N1} + C_{N2}x_{N2} + \cdots + C_{Nn}x_{Nn} = \sum_{j=1}^{n_N} c_{Nj}x_{Nj}$$

Where, C_{ij} is the eigen vector of elements of the largest eigen value of S, the covariance matrix of the components $x_{i1}, x_{i2}, \dots, x_{in_i}$. Thus, using the above equations, we get a new set of indicators $X_1, X_2, \dots, X_N$.

3.3 Objective Function

For building our objective function, we construct an index using the general formula-

$I = \sum_i w_i x_i$, where w_i is the weight assigned to x_i.

The calculation of these weights will be done using BWM

The steps of BWM measure for deriving weights of the objective function are as follows-

i) Determine the decision measure $\{d_1, d_2, \dots, d_p\}$.
ii) Determine the best and the worst measure.
iii) Determine the best-to-others (BO) and others-to-worst vector (OW) by assigning a value between 1 to 9, depending on the preference of the decision measure involved $V_B = (v_{B1}, v_{B2}, \dots, v_{Bp})$ and $V_W = (v_{1W}, v_{2W}, \dots, v_{pW}$; where, $v_{BB} = v_{WW} = 1$ (1 indicates equal preference).
iv) Find the optimal weights of the measure $({\omega_1}^*, {\omega_2}^*, \dots, {\omega_p}^*)$ such that the maximum of $\left|\frac{\omega_B}{\omega_j} - v_{Bj}\right|$ and $\left|\frac{\omega_j}{\omega_W} - v_{jW}\right|$ is minimized. The problem is formulated to a minimax model-

Minmax

$$\left\{\left|\frac{\omega_B}{\omega_j} - v_{Bj}\right|, \left|\frac{\omega_j}{\omega_W} - v_{jW}\right|\right\}_j$$

such that,

$$\sum_j \omega_j = 1$$

$\omega_j \geq 0,\ for\ all\ j$

which is equivalent to,

Min γ

Such that,

$\left|\frac{\omega_B}{\omega_j} - a_{Bj}\right| \leq \gamma\ for\ all\ j$

$\left|\frac{\omega_j}{\omega_W} - a_{jW}\right| \leq \gamma\ for\ all\ j$

$\sum_j \omega_j = 1$

$\omega_j \geq 0,\ for\ all\ j$

A comparison is said to be *fully consistent* if $v_{Bj}.v_{jW}$ =v_{BW} for all j.

For fully consistent comparison, we have $\gamma^* = 0$ and therefore we will have a unique optimal solution. If we find the minimax of $\{|\omega_B - v_{Bj}\omega_j|, |\omega_j - v_{jW}\omega_W|\}$ we get a linear model-

Minmax$_{\{\{|\omega_B - v_{Bj}\omega_j|, |\omega_j - v_{jW}\omega_W|\}\}_j}$

Such that

$\sum_j \omega_j = 1$

$\omega_j \geq 0,\ for\ all\ j$

which is equivalent to,

Min γ^L

Such that

$|\omega_B - v_{Bj}\omega_j| \leq \gamma^L\ for\ all\ j$

$|\omega_j - v\omega_W| \leq \gamma^L\ for\ all\ j$

$$\sum_j \omega = 1$$

3.4 Constraints

People from different social groups and regions put different emphasis on what is deemed important and gives more focus to activities that brings them joy. The dimension of life which a certain social group deems as important is its essence. It defines its uniqueness as a social group. We optimize the QOL of the given region by preserving its uniqueness. We build the constraint inequalities by ensuring that the essence of the given region is preserved and for this, we maintain at least the present state of quality of the dimension which is valued most by the people in the given system.

The first step for construction of constraints is grouping of the given system into homogeneous subsystems. In our study, the classification of the system into homogeneous subsystems is done according to the geographical area. We obtain a finite number of homogeneous subsystems say l number. In each subsystem S_r $(r = 1,2, \ldots, l)$, we represent the most valued variable by X_r.

Next, we set a particular $\alpha\epsilon[0,1]$. α is chosen according to the preferred precision level of linearity. Multiple regression analysis is then used to find the variables which are linearly influencing X_r with a significance level α. Regression means stepping back towards average. It is a statistical technique which gives the average strength of relationship between two or more variables based on their data. The predicted variable is named as the dependent variable and the predicting variable is named as the independent variable. When we have more than one variable as the independent variable, we use multiple regression. In multiple regression, the relationship between variables is given by $X = b_1Y_1 + b_2Y_2 + \cdots + b_nY_n + C$.

where, $\mathrm{b_1, b_2, \ldots, b_n}$ are the regression coefficients corresponding to $\mathrm{Y_1, Y_2, \ldots, Y_n}$ respectively.

X is the dependent variable. $\mathrm{Y_1, Y_2, \ldots, Y_n}$ are the independent variables. C is the constant.

In regression analysis, the overall statistical significance of the model is validated using F-test. The F-test determines whether the predicted variable is linearly related with the predicting variables or not. It employs a hypothesis test with $H_0: b_1 = b_2 = \cdots = b_n$ =0, as the null hypothesis and $H_1: b_i \neq 0$ for at leas one i, as the alternative hypothesis. If the probability of H_0 to be is less than the significance level that we have set (say α), we conclude that the dependent variable is linearly related to at least one of the independent variables. Consequently, we check which independent variable in particular, is related linearly with the dependent variable. In order to test the significance for each individual $Y_i(i = 1,2,..n)$, we use $H_0: b_i$ =0, as the null hypothesis and $H_1: b_i \neq 0$, as the alternative hypothesis. If the null hypothesis is significant with respect to the required α level, then we conclude that there is no significant linear relationship. And we remove Y_i from the regression model. For the formulation of our constraints, we set a particular significance level, say $\alpha\epsilon(0,1)$. Corresponding to this α, for each X_r, we find the variables which are influencing X_r using the regression equations-

$$\sum_{i\neq 1} b_iX_i + C_1 = X_1$$
$$\sum_{i\neq 2} b_iX_i + C_2 = X_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\sum_{i\neq l} b_iX_i + C_l = X_l$$

Considering our constraint, we have,

$$\underline{X_1} \leq \sum_{i\neq 1} b_iX_i + C_1 \leq \overline{X_1}$$
$$\underline{X_2} \leq \sum_{i\neq 2} b_iX_i + C_2 \leq \overline{X_2}$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\underline{X_l} \leq \sum_{i\neq l} b_iX_i + C_l \leq \overline{X_l}$$

3.5 Case Study

We have carried out a case study in Nagaland for the verification of the suggested model. Nagaland is a state in the northeastern region of India, a home to distinct indigenous tribes. It has 12 districts in total, namely Dimapur, Zunheboto, Wokha, Tuensang, Peren, Phek, Noklak, Mon, Mokokchung, Longleng, Kohima and Kiphire. Nagaland has an area of 16,579 square kilometers with a population of 1,980,602 according to the 2011 Census of India.

3.5.1 Selection of Initial QOL Indicators

Based on the theoretical assumptions, we have considered the following dimensions and the corresponding sub-dimensions-

i) **Standard of living (I_1):** Sub-dimensions: Income (I_{11}), Indebtedness (I_{12}), Accessto electronics (I_{13}), Food sufficiency (I_{14}), Ability to support education (I_{15}), Sanitary condition (I_{16}), Sufficiency of space/rooms in house (I_{17}), Ownership of house (I_{18}), Surrounding of house (I_{19}).

ii) **Employment and job (I_2):** Sub-dimensions: State of employment (I_{21}), Healthy environment of workplace (I_{22}), Relation with colleagues (I_{23}), Job satisfaction (I_{24}), Workplace safety (I_{25}), Family employment rate (I_{26}).

iii) **Health (I_3):** Sub-dimensions: Body Mass Index (BMI) (I_{31}), Forgetfulness (I_{32}), Frequency of getting headaches (I_{33}) , Vision (I_{34}), Problem related to walking/movement (I_{35}), Lower abdominal pain (I_{36}), Surgeries done (I_{37}), Frequency of getting hospitalized (I_{38}), Sleeplessness (I_{39})Difficulties in carrying out tasks (I_{310}), Depression (I_{311}),Psychological health (I_{312}), Family rate of life-threatening health issues (I_{313}), Quality of hospitals (I_{314}).

iv) **Education (I_4):** Sub-dimensions: Quality of educational institutions (I_{41}), Access to good library (I_{42}), Students from other states/countries (I_{43}), Extracurricular activities (I_{44}), Knowledge and skills of computers (I_{45}).

v) **Social wellbeing (I_5):** Sub-dimensions: Frequency of attending social gatherings (I_{51}), Contribution to society (I_{52}), Relation with neighbours (I_{53}), Security status in society (I_{54}), Contribution to sports (I_{55}), Having trustworthy friends or relatives (I_{56}), How often one catches up with friends and relative (I_{57}).

vi) **Governance and basic rights (I_6):** Sub-dimensions: Satisfaction with present government (I_{61}), Law system(I_{62}), Transportation system (I_{63}), Gender bias (I_{64}), Ethical standard of politicians (I_{65}), Efficiency of public services (I_{66}).

vii) **Environment (I_7):** Sub-dimensions: Access to fresh drinking water (I_{71}), Natural disasters (I_{72}), Transportation system (I_{73}), Health issues due to environment pollution (I_{74}), Rate of tree plantation (I_{75}), Density of greenery (I_{76}), Waste management. (I_{77}).

3.5.2 Data Collection and Pre-Processing

For each of the sub-dimensions discussed above, we have collected the data subjectively through questionnaires. The total population of Nagaland as per 2011 census is 19.79 lakhs. We have collected data for 300 households. The sample size is comparatively small, owing to a low budget. The stratified random sampling scheme has been employed for a better coverage of the data, taking into consideration, the heterogeneity of the characteristics under study. 25 samples from each of the 12 districts of Nagaland have been taken. For pre-processing of data, ratings in a range [0, 1], for each of the sub-dimensions has been assigned according to the response in the questionnaire and the subjective data is converted into numerical data.

3.5.3 Selection of Final QOL Indicators

The selected variables have shown sufficient correlation with a high KMO (Kaiser-Meyer-Olkin) measure of sampling adequacy- 0.683 and a low probability associated with Bartlett test of sphericity- 0.000. Thus, PCA can be applied. After applying PCA, we arrive to a set of indicator variables that explains most of the variance in the dataset.

Table 1: Total Variance Explained

Component	Initial Eigen values			Eigen values of extracted components			Eigen values of extracted rotated components		
	Total	% of variance	Cumulative %	Total	% of variance	Cumulative %	Total	% of variance	Cumulative %
1	18.570	64.815	64.815	18.570	64.815	64.815	0.893	3.118	3.118
2	2.122	7.405	72.220	2.122	7.405	72.220	2.195	7.660	10.778
3	2.056	7.176	79.396	2.056	7.176	79.396	3.042	10.616	21.394
4	2.024	7.064	86.460	2.024	7.064	86.460	2.059	7.186	28.580
5	0.836	2.919	89.379	0.836	2.919	89.379	17.419	60.799	89.379
6	0.382	1.332	90.711						

It is observed that only a few numbers of the initially selected indicators are significant for final selection for the study. This reflects the social and economic uniqueness of Nagaland from the rest of the world. A total of 5 components have been extracted- Employment (X_1 with 5 parameters), Mental health (X_2 with 1 parameter), House-ownership (X_3 with 1 parameter), Clothing(X_4 with 1 parameter), Electricity supply (X_5 with 1 parameter). A total of 89.39% of variance is explained.

3.5.4 Objective Function

In the current case, the first step in BWM to select the decision criteria is already done using PCA. We have selected X_1(Employment), X_2 (Mental health), X_3(House-ownership), X_4(Clothing) and X_5(Electricity supply). Among all the criteria, the best and worst were selected and BO, OW vectors were determined by experts through mutual consensus. The linguistic scale used for determining the pair wise desirability is given as below-

Table 2: Linguistic Scale

Equally desirable	1
Almost equally desirable	2
Moderately desirable	3
Moderate to Strongly desirable	4
Strongly desirable	5
Strongly to Very Strongly desirable	6
Very strongly desirable	7
Extremely desirable	8
Perfectly desirable	9

Table 3: Best-to-others vector

BO	X_1	X_2	X_3	X_4	X_5
Best Criterion: Employment, X_1	1	6	3	3	2

Thus, from Table 3 we observe that, $V_B = (1,6,3,3,2)$. X_1 being the best criteria, has a value 1. X_1 is strongly to very strongly desirable over X_2, which has a value 6. X_1 is moderately desirable over X_3 and X_4 as both have same value 3. X_1 is almost equally desirable over X_5 as it has a value 2.

Table 4: Others-to-Worst vector

WO	X_1	X_2	X_3	X_4	X_5
Best Criterion: Employment, X_1	6	1	2	2	3

From Table 4, it may be confirmed that, $V_W = (6,1,2,2,3)$. X_2 being the worst criteria has a value 1. X_1 is strongly to very strongly desirable over X_2as it has a value 6. $X_3$and X_4 are almost equally desirable over X_2, since both have values 2. X_5 is moderately desirable over $X_2$and has a value 3.

Solving the following linear model-

min γ^L

s.t.

$$|\omega_B - 1\omega_1| \le \gamma^L,\ |\omega_1 - 6\omega_W| \le \gamma^L,\ |\omega_B - 6\omega_2| \le \gamma^L,\ |\omega_2 - 1\omega_W| \le \gamma^L$$
$$|\omega_B - 3\omega_3| \le \gamma^L,\ |\omega_3 - 2\omega_W| \le \gamma^L,\ |\omega_B - 3\omega_4| \le \gamma^L,\ |\omega_4 - 2\omega_W| \le \gamma^L$$
$$|\omega_B - 2\omega_5| \le \gamma,\ |\omega_5 - 3\omega_W| \le \gamma^L,\ \textstyle\sum_j \omega_j = 1$$
$$\omega_j \ge 0,\ \text{for all j}$$

We get $\omega_1 = 0.429,\ \omega_2 = 0.071,\ \omega_3 = 0.143,\ \omega_4 = 0.143,\ \omega_5 = 0.214$ and $\xi^L = 0$.

Thus, the objective function of QOLI is-

$$I = 0.429X_1 + 0.071X_2 + 0.143X_3 + 0.143X_4 + 0.214X_5$$

3.5.5 Constraint Equations

We group the given system into homogeneous subsystems based on the geographical area. In the present study, we have 12 districts namely Mon (D_1), Phek (D_2), Kiphire (D_3), Noklak (D_4), Kohima (D_5), Dimapur (D_6), Mokokchung (D_7), Peren (D_8), Tuensang (D_9), Wokha (D_{10}), Zunheboto (D_{11}) and Longleng (D_{12}). From the collected data, it is observed that the people of districts D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_{11} and D_{12} wants to preserve variable X_2(mental health). People of District D_8 and D_{10} wants to preserve X_5(Electricity supply). People of District D_9 wants to preserve X_3(House ownership). Therefore, $h_r = X_2$ for $r = 1, 2, 3, 4, 5, 6, 7, 11, 12$. $h_r = X_5$ for r =8, 10. We set a particular significance level, $\alpha = 0.2$ and apply multiple regression analysis.

Table 5: Regression Results for D_1

Coefficients				
	β	ε_s	t	Significance
(Constant)	0.754	0.135	5.592	0.000
X_3	0.359	0.169	2.118	0.046
X_4	-0.379	0.286	-1.324	0.199

For district D_1, X_2(mental health) is the dependent variable and from the regression analysis it is observed that X_3 has a positive influence on X_2 with a value of 0.359 whereas X_4 has a negative influence of -0.379. The significance value for both X_3 and X_4 is less than 0.2.

Table 6: Regression Results for D_2

Coefficients				
	β	ε_s	t	Significance
(Constant)	0.772	0.062	12.456	0.000
X_3	-0.239	0.105	-2.269	0.033

For district D_2, X_2(mental health) is the dependent variable and from the regression analysis it is observed that X_3 has a negative influence on X_2 with a value -0.239. The significance value is 0.033 which is less than 0.2.

Table 7: Regression Results for D_3

Coefficients				
	β	ε_s	t	Significance
(Constant)	0.804	0.122	6.598	0.000
X_4	-0.431	0.291	-1.481	0.152

For district D_3, X_2(mental health) is the dependent variable and from the regression analysis it is observed that X_4 has a negative influence on X_2 with a value -0.431 and the significance value for X_4 is less than 0.2.

Table 8: Regression Results for D_4

Coefficients				
	β	ε_s	t	Significance
(Constant)	0.829	0.061	13.499	0.000
X_1	-0.263	0.125	-2.108	0.046

For district D_4, X_2(mental health) is the dependent variable and from the regression analysis it is observed that X_1 has a negative influence on X_2 with a value -0.2108 and the significance value is less than 0.2.

Table 9: Regression Results for D_5

Coefficients				
	β	ε_s	t	Significance
(Constant)	0.504	0.112	4.481	0.000
X_3	0.276	0.182	1.515	0.143

For district D_5, X_2(mental health) is the dependent variable and from the regression analysis it is observed that X_3 has a positive influence on X_2 with a value 0.276 and the significance value is less than 0.

Table 10: Regression Results for D_7				
Coefficients				
	β	ε_s	t	Significance
(Constant)	0.743	0.062	11.920	0.000
X_2	-0.206	0.116	-1.772	0.090

For district D_7, X_3(house ownership) is the dependent variable and from the regression analysis it is observed that X_2 has a negative influence on X_3 with a value -0.206 and the significance value for $X_2$is less than 0.2.

Table 11: Regression results for D_8				
Coefficients				
	β	ε_s	t	Significance
(Constant)	0.377	0.143	2.630	0.015
X_4	0.519	0.304	1.706	0.101

For district D_8, X_5(electricity supply) is the dependent variable and from the regression analysis it is observed that X_4 has a positive influence on X_5 with a value 0.519 and the significance value for $X_4$is less than 0.2.

Table 12: Regression Results for D_9				
Coefficients				
	β	ε_s	t	Significance
(Constant)	0.389	0.107	3.636	0.001
X_1	0.287	0.181	1.584	0.127

For district D_9, X_3(house ownership) is the dependent variable and from the regression analysis it is observed that X_1 has a positive influence on X_3 with a value 0.287 and the significance value for $X_1$is less than 0.2.

Table 13: Regression Results for D_{11}				
Coefficients				
	β	ε_s	t	Significance
(Constant)	0.426	0.145	2.934	0.007
X_4	0.660	0.338	1.955	0.063

For district D_{11}, X_2(mental health) is the dependent variable and from the regression analysis it is observed that X_4 has a positive influence on X_4 with a value 0.660. The significance value for $X_4$is 0.063 which is less than 0.2.

Table 14: Regression results for D_{12}				
Coefficients				
	β	ε_s	t	Significance
(Constant)	0.581	0.128	4.539	0.000
X_5	-0.386	0.205	-1.884	0.073
X_3	0.265	0.153	1.731	0.098

For district D_{12}, X_2(mental health) is the dependent variable and from the regression analysis it is observed that X_3 has a positive influence on X_2 with a value 0.265 whereas X_5 has a negative influence of -0.386. The significance value for both X_3 and X_4 is less than 0.2.

We take a note that for district D_6 and D_{10}, there are no significant variables which have linear influence on the dependent variable. The variable "B" in the table represents the unstandardized beta value. The t-value and the p-value are represented as "t" and "Sig." respectively.
Thus, we have the constraints for $\alpha = 0.20$ as-

$0.25 \leq 0.359\, X_3 - 0.379\, X_4 + 0.754 \leq 1$, $0.25 \leq -0.239\, X_3 + 0.772 \leq 1$, $0 \leq -0.431 X_4 + 0.804 \leq 1$,
$0.25 \leq -0.263 X_1 + 0.829 \leq 1$, $0 \leq 0.276\, X_3 + 0.504 \leq 1$, $0 \leq 0.519\, X_4 + 0.377 \leq 1$,
$0.25 \leq -0.206\, X_1 + 0.743 \leq 1$, $0 \leq 0.287\, X_1 + 0.389 \leq 0{,}75$, $0.25 \leq 0.660\, X_4 + 0.426 \leq 1$,
$0 \leq -0.386\, X_5 + 0.265\, X_3 + 0.581 \leq 1$.

3.5.6 QOLI Optimization Model for Nagaland

For $\alpha = 0.20$:
Maximize, I = $0.429\, X_1 + 0.071\, X_2 + 0.143\, X_3 + 0.143\, X_4 + 0.214\, X_5$
Subject to the constraints,
$0.25 \leq 0.359\, X_3 - 0.379\, X_4 + 0.754 \leq 1{,}25 \leq -0.239\, X_3 + 0.772 \leq 1{,}0 \leq -0.431 X_4 + 0.804 \leq 1$,
$0.25 \leq -0.263 X_1 + 0.829 \leq 1$, $0 \leq 0.276\, X_3 + 0.504 \leq 1{,}0 \leq 0.519\, X_4 + 0.377 \leq 1{,}0.25 \leq -0.206\, X_1 + 0.743 \leq 1$, $0.287\, X_1 + 0.389 \leq 0.75$, $0.25 \leq 0.660\, X_4 + 0.426 \leq 1$, $0 \leq -0.386\, X_5 + 0.265\, X_3 + 0.581 \leq 1$,
$0 \leq X_i \leq 1$, $\forall\, i = 1,2,3,4,5,6,7$

After solving the above achieved Linear Programming Problem by Excel Solver we have obtained following results.

3.5.7 Result and Analysis

Table 15: Result

X_1	X_2	X_3	X_4	X_5
1	1	1	0.869	0.002

In Nagaland, for α =0.20, if we want to reach the optimum QOLI of 0.768 from a maximum of 1. We need to maintain a rating as high as 1 for Employment (X_1), Mental health (X_2) and house ownership (X_3), while maintaining a comparatively lower rating of 0.869 and 0.002 for Clothing (X_4) and Electricity supply(X_5) respectively. Employment(X_1), although it shows some negative relation with mental health(X_2), it has a high value due to the high preference in the objective function. Mental health(X_2), being the variable, which is valued by majority of the districts, has a value 1. House ownership(X_3), is the variable which is valued most by the people of the district 9 and also shows positive impact on mental health(X_2), therefore it has a high rating. Clothing (X_4) has a little lower optimal rating since it shows some negative impact on the most valued variableX_2, also it is not the most valued variable for any of the districts. Electricity Supply (X_5) shows only negative impact towards the valued variable(X_2) and so it has a very low optimal value.

4. CONCLUSION

A technique for formulating a Linear Programming Problem for optimum Quality of Life Index has been discussed. The proposed method incorporates statistical ideas with operational research concepts to find the optimal QOLI of a particular region. The method, to a great extent, broadens the real-life applicability of LPP. Although BWM is a dynamic method, better method for weight assignment would help in strengthening the QOLI optimization model. The discussed method is confined not only to QOLI optimization but also it can be applied for building similar models which exhibit an approximate nature of linearity, with or without the requirement of slight modifications. In the case study of Nagaland state, it is observed that Employment, Mental health of inhabitant as well as House ownership are making maximum contribution in excelling the QOL index in this developing state. Focusing on improving these factors would immensely help in improving the QOL of the people in Nagaland, India This mathematical modelling procedure has lot of scope in the formation of various other social indexing such as optimum quality of education index, optimum quality of health index etc. to support the decision maker in the development of the particular area.

REFERENCES

Costanza R, Fisher B, Ali S, Beer C, Bond L, Boumans R, et al.(2007). Quality of life: An approach integrating opportunities, human needs, and subjective well-being. *Ecological Economics*; 61, 267–76. https://doi.org/10.1016/J.ECOLECON.2006.02.023.

Diener E, Suh E.(1997) Measuring Quality Of Life: Economic, Social, And Subjective Indicators. *Social Indicators Research* 40:1 1997; 40:189–216. https://doi.org/10.1023/A:1006859511756.

Durand M. (2015) The OECD Better Life Initiative: How's Life? and the Measurement of Well-Being. Review of Income and Wealth 2015; 61, 4–17. https://doi.org/10.1111/ROIW.12156.

Jolliffe IT.(1973) Discarding Variables in a Principal Component Analysis. Ii: Real Data. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*; 22, 21–31. https://doi.org/10.2307/2346300.

Koronakos G, Smirlis Y, Sotiros D, Despotis DK.(2020) Assessment of OECD Better Life Index by incorporating public opinion. Socio-Economic Planning Sciences 2020; 70, 100699. https://doi.org/10.1016/J.SEPS.2019.03.005.

Ma L, Liu S, Fang F, Che X, Chen M.(2020) Evaluation of urban-rural difference and integration based on quality of life. *Sustainable Cities and Society*; 54, 101877. https://doi.org/10.1016/J.SCS.2019.101877.

Omrani H, Alizadeh A, Amini M. (2020) A new approach based on BWM and MULTIMOORA methods for calculating semi-human development index: An application for provinces of Iran. *Socio-Economic Planning Sciences*; 70, 100689. https://doi.org/10.1016/J.SEPS.2019.02.004.

Põldaru R, Roots J. (2014) A PCA–DEA approach to measure the quality of life in Estonian counties. *Socio-Economic Planning Sciences*; 48, 65–73. https://doi.org/10.1016/J.SEPS.2013.10.001.

Pun CS, Wong HY.(2019) A linear programming model for selection of sparse high-dimensional multiperiod portfolios. *European Journal of Operational Research*; 273, 754–71. https://doi.org/10.1016/J.EJOR.2018.08.025.

Rezaei J. (2016) Best-worst multi-criteria decision-making method: Some properties and a linear model. *Omega* 2016; 64, 126–30. https://doi.org/10.1016/J.OMEGA.2015.12.001.

Rezaei J, van Roekel WS, Tavasszy L.(2018) Measuring the relative importance of the logistics performance index indicators using Best Worst Method. *Transport Policy* 2018; 68, 158–69. https://doi.org/10.1016/J.TRANPOL.2018.05.007.

Slottje DJ. (1991) Measuring the Quality of Life Across Countries. *The Review of Economics and Statistics* 1991; 73:684. https://doi.org/10.2307/2109407.

Stiglitz J, Sen A, Fitoussi J. (2009) Report by the commission on the measurement of economic performance and social progress 2009.

Tripathi M, Singal SK. (2019) Use of Principal Component Analysis for parameter selection for development of a novel Water Quality Index: A case study of river Ganga India. *Ecological Indicators* 2019; 96, 430–6. https://doi.org/10.1016/J.ECOLIND.2018.09.025.

Wang T, Fu Y.(2019) Constructing Composite Indicators with Individual Judgements and Best–Worst Method: An Illustration of Value Measure. *Social Indicators Research* 2019 149:1 2019; 149,1–14. https://doi.org/10.1007/S11205-019-02236-3.

Zhu J. (2001) Multidimensional quality-of-life measure with an application to Fortunes best cities. Socio-Economic Planning Sciences; 35, 263–84. https://doi.org/10.1016/S0038-0121(01)00009-X.

Zhu F, Ukkusuri S v. (2015) A linear programming formulation for autonomous intersection control within a dynamic traffic assignment and connected vehicle environment. *Transportation Research Part C: Emerging Technologies* 2015; 55, 363–78. https://doi.org/10.1016/J.TRC.2015.01.006.