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PERFORMANCE ANALYSIS OF CLOSED TWO NODE QUEUEING NETWORK WITH PREEMPTIVE PRIORITY

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ABSTRACT

In this paper, we analyze closed two node queueing network with preemptive different priority disciplines at both nodes. The service times follow negative exponential distribution for both low and high priority customers. The governing equations of the model are provided in section 3. The equilibrium distribution is obtained for both preemptive different and preemptive repeat identical priority disciplines by assuming state dependent rates in sections 4 and 5. The results for preemptive resume discipline for both state dependent and constant rates are also obtained. In section 6, we deduce some special cases. The steady state probability vector is derived by implementing matrix-geometric approach in section 7. In section 8, the applicability of preemptive priority in practical life situations is highlighted.

Keywords : Two node queueing network, preemptive priority, preemptive repeat identical priority, steady state probability vector, metric-geometric approach.

1. INTRODUCTION

Service priorities have direct impact on the service performance and capacity utilization of the operation of almost all the manufacturing/service system. In particular, priority is probably used to allocate limited production capacity among jobs with different needs and willingness to pay. The most general situation in priority disciplines in preemptive priority in which the job with highest priority are allowed to enter service immediately suspending the service of job with lower priority, which is already in service. According to preemptive repeat different discipline, the preempted unit on its re-entry requires a random service time independent of past preemptions and wasted service time. The queueing network is frequently used to model a computer system which consists of a central processing unit (CPU) and an input/output (I/O) device. The computer system processes both interactive (high priority) and batch (low priority) jobs.

2. RELATED REVIEW

Queueing models with priority have drawn the attention of several research workers due to its applications in many congestion situations encountered in production, manufacturing, distribution systems, etc. Sivasamy (1986) discussed a queueing model with single server facility in which the units having preemptive priority are served by using bulk service rule. Li et al. (1989) obtained Laplace-Stieltjes transform of the queue size distribution and the waiting time distributions of a stationary process for a priority queueing model of a production system. Katayama (1992) derived queue-length generating functions and Laplace - Stieltjes transforms of waiting time distributions for two class priority queue.

Wagner (1998) derived the steady state distributions immediately after arrival instants of the different priority classes by using matrix geometric methods for a non-preemptive head-of-the line multi-server multi-queue priority model with finite buffer capacity for each priority class. Drekic and Stanford (2000) determined the thresholds that optimize performance measures such as overall average sojourn time etc. for single-server priority queueing model with preemptive resume and preemptive repeat service disciplines. Chen and Ye (2001) established a new sufficient condition for the existence of the diffusion approximation for multi-class queueing networks under preemptive resume priority service discipline. Bitran and Caldentey (2002) presented a performance analysis of a 2-dimensional preemptive priority queueing system with state dependent arrivals.

Several researchers have provided analysis of various queueing models by using matrix geometric method. Neuts (1981) provided systematic and detailed infrastructure about matrix analytic and matrix geometric method of various stochastic models in his textbook. Lucantoni and Ramaswami (1985) analyzed phase type queueing models and provided algorithms for solving the non-linear matrix equations. Buzacott and Kostelski (1987) presented matrix geometric and recursive algorithm for a two stage unreliable flow line. Sengupta (1990) provided phase type representation for matrix-geometric solution. Krishna Reddy et al. (1993) obtained steady state probability vectors of the number of customers in the queue, the stability condition and mean queue length of customers by using the matrix geometric method for a multi-server non-preemptive priority queueing system. Jayaraman et al. (1994) obtained steady-state probability vector of the number of customers in the queue and the stability condition by using a matrix geometric algorithm approach for a general bulk service queue. Haverkort and Ost (1997) provided comparison between spectral expansion and matrix-geometric method for steady state analysis of Petri nets. Akar et al. (1998) approximated matrix-geometric solutions for M/G/1 type markov chains. Gomez (2002) investigated tandem queues with blocking and repeated attempts by implementing matrix-geometric approximation. Grey et al. (2002) analyzed queueing models with backup servers and services breakdowns. They implemented matrix-geometric approach to determine the queue distribution. Choi et al. (2003) derived matrix-geometric solution for nested quasi birth-death chains. Harchol-Balter et al. (2005) made a significant step in the analysis of the general case of a priority multi-service queue by considering servers that combine both non-exponential service times and non-identical service rates over all the classes. Boltch et la. (2006) discussed various aspects of queueing systems of multi-priority queues. Gupta et al. (2007) derived the accurate approximate analysis of a queue with multiple servers and general service times without priorities. Zeltyn et al. (2007) derived waiting and sojourn times in a multi-service queue with mixed priorities. Ellens et al. (2012) discussed about performance of cloud computing centres with multiple priority queues. Lin et al. (2014) estimated the waiting time of multi-priority emergency patients with downstream blocking. Wang et al. (2015) discussed the results about M/MC queue with two priority classes. Hanbali et al. (2015) approximated the waiting time distribution in M/Ph/C priority queue. Ammar and Rajadurai (2019) investigated the impact of disaster on the retrial queueing system by including the features of the preemptive priority and working breakdown server.

3. THE MODEL AND GOVERNING EQUATIONS

We consider a closed queueing networks with preemptive priority at both left and right nodes. We present the analytical results for preemptive different, preemptive repeat identical and preemptive resume disciplines for two node closed queueing network.
The assumptions related to queueing network are given as follows.

- The system consists of N high and M low priority customers. The customers may neither join nor leave the network.
- The complete description of the state of the system is given by (n,m) where n and m demote the number of high and low priority customers situated at the left node.
- The service time for high and low priority customers is distributed negative exponentially with parameters □$_i(n,m)$ and □$_i(n,m)$ respectively.
- p_i is the probability that the customer is in state i and $q\ (i, j)$ is rate of moving from state i to state j.
- $p_{i,j}$ be the equilibrium distribution of the system when there are i high and j low priority customers.
- High priority customers have preemptive different priority discipline over low priority customers at each node.
- The system is cyclic, i.e. the customers pass to the other queue immediately upon completion of service.

The steady state equations governing the model are as follows:

$$p_{00}\left(\mu_2+\sigma_2\right)-p_{10}\sigma_1-p_{01}\mu_1=0 \tag{1}$$

$$p_{i0}\left(\mu_2+\sigma_1+\sigma_2\right)-p_{(i+1)0}\,\sigma_1-p_{(i-1)0}\sigma_2=0\ ;\ i=1,2,\ldots,N-1 \tag{2}$$

$$p_{N0}\left(\sigma_1+\mu_2\right)-p_{(N-1)0}\sigma_2=0 \tag{3}$$

$$p_{01}(\mu_1+\mu_2+\sigma_2)-p_{02}\mu_1-p_{00}\mu_2-p_{11}\sigma_1=0 \quad (4)$$

$$p_{ij}(\sigma_1+\sigma_2+\mu_2)-p_{(i+1)j}\sigma_1-p_{(i-1)j}\sigma_2-p_{i(j-1)}\mu_2=0 \quad (i=1,2,\ldots,N-1,\ j=1,2,\ldots,M-1) \quad (5)$$

$$p_{Nj}(\sigma_1+\mu_2)-p_{(N-1)j}\sigma_2-p_{N(j-1)}\mu_2=0 \quad ; j=1,2,\ldots,M-1 \quad (6)$$

$$p_{0M}(\sigma_2+\mu_1)-p_{1M}\sigma_1-p_{0(M-1)}\mu_2=0 \quad (7)$$

$$p_{iM}(\sigma_1+\sigma_2)-p_{(i+1)M}\sigma_1-p_{(i-1)M}\sigma_2-p_{i(M-1)}\mu_2=0 \quad (8)$$

$$p_{NM}(\sigma_1)+p_{(N-1)M}\sigma_2-p_{N(M-1)}\mu_2=0 \quad (9)$$

4. THE MODEL WITH PREEMPTIVE DIFFERENT PRIORITY DISCIPLINE

According to preemptive different discipline, a random service time independent of past preemptions and wasted service time is required for the preempted customer on its reentry. The state transition rate diagram for two node preemptive different priority system is shown in figure 1.

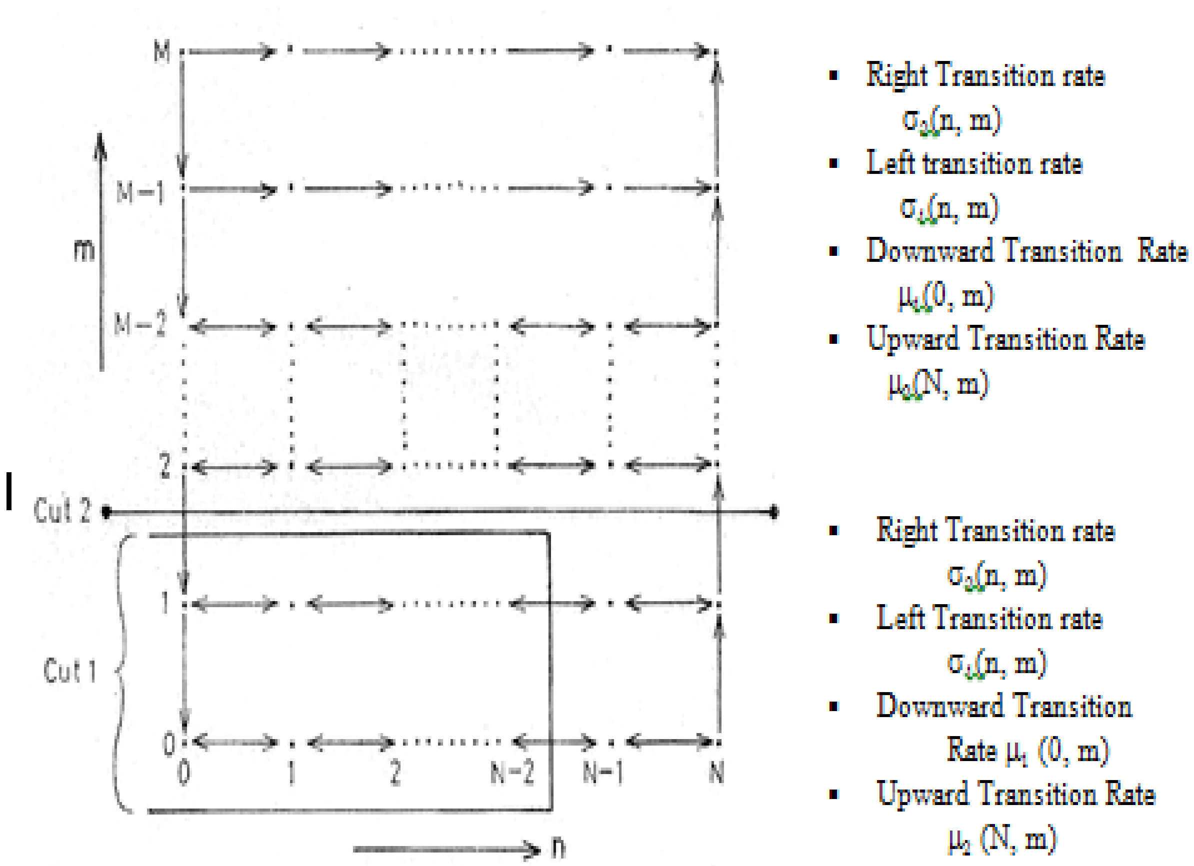

Figure 1: State transition diagram for two node preemptive different priority system.

The transition rates are independent of the system state, i.e. $\sigma_i(n,m)=\sigma_i, \mu_i(n,m)=\mu_i \forall n,m.$ We derive global balance equations for the queueing network. By using these equations, we obtain second order homogeneous recurrence relations by which the solution is obtained.

For deriving the balance equations, we equate the flux into any set S by the flux out of the set.

i.e. $$\sum_{i\in S}\sum_{j\in S_1} p_i q(i,j) = \sum_{j\in S_1}\sum_{i\in S} p_j q(j,i) \qquad (10)$$

I. The Equations for Cut 1

We obtain global balance equations corresponding to cut 1 with the help of state transition rate diagram shown in figure 1.

Let us consider that set $S = \{(i,m); 0 \le i \le n < N\}$. Then

$$p_{n,m}\sigma_2 + p_{0,m}\mu_1(1-\delta_{0\,m}) = p_{n+1,m}\sigma_1 + p_{0,2m}\mu_1(1-\delta_{m\,M}) \qquad (11)$$

Education (11) can be rewritten as

$$p_{n+1,m}\,\sigma_1 - p_{n,m}\,\sigma_2 = \mu_1\{p_{0,m}(1-\delta_{0m}) - p_{0,2m}(1-\delta_{m\,M})\} \qquad (12)$$

where δ_{ij} is the kronecker delta.

By using recursive method, the solution of equations (1-9) is given by

$$p_{n,m} = p_{0,m}\left(\frac{\sigma_2}{\sigma_1}\right)^n + \left(\frac{\mu_1}{\sigma_1}\right)\{p_{0,m}(1-\delta_{0m}) - p_{0,2m}(1-\delta_{m\,M})\}\left\{\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^n}{1-\frac{\sigma_2}{\sigma_1}}\right\} \qquad (13)$$

For complete solution for $p_{n,m}$, we must have a relationship between $p_{0,m}$ and $p_{0,2m}$.

II. The Equations for Cut 2

To obtain such relationship, we derive balance equation corresponding to another cut 2.

Let us consider set $S = \{(i,j), 0 \le i \le N, 0 \le j \le m\}\ m = 0,1,\ldots\ldots,M-1$. Then $p_{0,2m}\,\mu_1 = p_{N,m}\mu_2$ (14)

Using equation (13), we get the value of $p_{N,m}$ as

$$p_{N,m} = p_{0,m}\left(\frac{\sigma_2}{\sigma_1}\right)^N + \left(\frac{\mu_1}{\sigma_1}\right)\{p_{0,m}(1-\delta_{0m}) - p_{0,2m}(1-\delta_{mM})\}\left\{\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^N}{1-\left(\frac{\sigma_2}{\sigma_1}\right)}\right\} \qquad (15)$$

By putting the value of $p_{N,m}$ in equation (5), we obtain the result for $p_{0,2m}$ as

$$p_{0,2m} = \frac{1}{B_m}\left[\left(\frac{\mu_2}{\mu_1}\right)p_{0,m}\left(\frac{\sigma_2}{\sigma_1}\right)^N + \left(\frac{\mu_2}{\sigma_1}\right)\{p_{0,m}(1-\delta_{0m})\}\left\{\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^N}{1-\frac{\sigma_2}{\sigma_1}}\right\}\right] \qquad (16)$$

where $$B_m = 1 + \frac{\mu_2}{\sigma_1}(1-\delta_{m\,M})\left\{\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^N}{1-\left(\frac{\sigma_2}{\sigma_1}\right)}\right\} \qquad (17)$$

The complete solution for $p_{n,m}$ is obtained by substituting the value of $p_{0,2m}$ in equation (13) as

$$p_{n,m} = p_{0,m}\left(\frac{\sigma_2}{\sigma_1}\right)^n + p_{0,m}\left(\frac{\mu_1}{\sigma_1}\right)\left(1-\delta_{0,m}\right)\left[1-\frac{1}{B_m}\left\{\begin{array}{l}\left(\frac{\mu_2}{\mu_1}\right)\left(\frac{\sigma_2}{\sigma_1}\right)^N \\ +\left(\frac{\mu_2}{\sigma_1}\right)\left(\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^N}{1-\frac{\sigma_2}{\sigma_1}}\right)\left(\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^N}{1-\left(\frac{\sigma_2}{\sigma_1}\right)}\right)\left(1-\delta_{mM}\right)\end{array}\right\}\right] \quad (18)$$

The normalizing equation $\sum_{n=0}^{N}\sum_{m=0}^{M} p_{n,m} = 1$ is used to obtain $P_{0,0}$.

4.1 Preemptive Different Discipline Model with State Dependent Rates

Let the service times for high and low priority customers be distributed negative exponentially with parameters $\sigma_i(n,m)$ and $\mu_i(n,m)$ respectively. In this case, the equilibrium distribution $p_{n,m}\{0 \le n \le N, 0 \le m \le M\}$ is obtained by using product form solution. Thus,

$$p_{0,m} = A_m K(j) \quad (19)$$

where $A_m = p_{1,0}\ \sigma_2(1,0) + p_{1,0}\ \mu_2(1,0) - p_{2,0}\,\sigma_1(2,0)$.

$$K(j) = 1 + \sum_{i=1}^{n-1}\prod_{j=1}^{i}\frac{\mu_2(n-j,m)}{\mu_1(n-j,m)} + \sum_{i=1}^{n-1}\prod_{j=1}^{i}\frac{\sigma_2(n-j,m)}{\mu_1(n-j,m)}$$

Thus, $$p_{n,m} = A_m K(j)\left[p_{0,0}\ \mu_2\ (0,0)\prod_{i=0}^{n-1}\frac{\mu_2(i,m)}{\mu_1(i,m)} - \sum_{i=1}^{n-1} p_{i,i}\sigma_i(i,1)\right] \quad (20)$$

5. PREEMPTIVE REPEAT IDENTICAL DISCIPLINE MODEL

The equilibrium distribution for the system with preemptive repeat identical discipline at both nodes is obtained by deriving global balance equations in the system.

According to preemptive repeat identical discipline, the preempted customer requires the same amount of service after reentry as it required on its earlier entry.
Let us consider set $\delta = \{(i,m); 0 \le i \le n \le N\}$
The balance equation in the system is given by

$$p_{n,m}\sigma_2 + p_{0,m}\ \mu_1(1-\delta_{0m}) = p_{n+1,m}\sigma_1 + p_{0,m}\ \mu_1(1-\delta_{mM}) \quad (21)$$

The above equation can be rewritten as

$$p_{n+1,m}\sigma_1 - p_{n,m}\sigma_2 = \mu_1\left[p_{0,m}(1-\delta_{0m}) - p_{0,m}(1-\delta_{m,M})\right] \quad (22)$$

where δ_{ij} the kronecker delta.
By using recursive solution technique, we obtain the equilibrium distribution $p_{n,m}$ as follows:

$$p_{n,m} = p_{0,m}\left(\frac{\sigma_2}{\sigma_1}\right)^n + p_{0,m}\left(\frac{\mu_1}{\sigma_1}\right)(\delta_{mM} - \delta_{0m})\left\{\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^n}{1-\frac{\sigma_2}{\sigma_1}}\right\} \tag{23}$$

and $p_{n0} = p_{00}\left(\frac{\sigma_2}{\sigma_1}\right)^n$.

p_{00} can be obtained by using normalizing condition $\sum_{n=0}^{N}\sum_{m=0}^{M} p_{n,m} = 1.$

6. SPECIAL CASES

Case 1 : Preemptive Resume Discipline and Constant Rates

In case of preemptive resume discipline at both nodes of the system, the equilibrium distribution $p_{n,m}$ is given by

$$p_{n,m} = p_{00}H^{m-1}\left[\left(\frac{\sigma_2}{\sigma_1}\right)^n + \left(\frac{\mu_1}{\sigma_1}\right)\left\{1 - H(1-\delta_{mM})\left(\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^n}{1-\frac{\sigma_2}{\sigma_1}}\right)\right\}\right] \tag{24}$$

and $$p_{0,0} = \left[\left\{\left(\frac{1-H^M}{1-H}\right)+1\right\}\left\{\frac{1-\left(\frac{\sigma_2}{\sigma_1}\right)^{N+1}}{1-\frac{\sigma_2}{\sigma_1}}\right\}\right]^{-1} \tag{25}$$

where $$H = \frac{(\sigma_2/\sigma_1)^N}{\mu_2/\mu_1 + \mu_1\left\{1-\left(\frac{\sigma_2}{\sigma_1}\right)^N\right\} /(\sigma_1 - \sigma_2)}$$

Case 2 : Preemptive Resume Discipline and State Dependent Rates

If the system follows preemptive resume discipline at both nodes of the system and the service times for high and low priority customers are distributed negative exponentially with state dependent parameters $\sigma_i(n,m)$ and $\mu_i(n,m)$ respectively, then the equilibrium distribution $p_{n,m}\{0 \le n \le N, 0 \le m \le M\}$ is given by

$$p_{0,m} = p_{0,0}\prod_{j=0}^{m-1} K(j) \quad ; \quad 1 \le m \le M \tag{26}$$

$$p_{n,m} = p_{0,0}\prod_{j=0}^{m-1} K(j)\left[\prod_{i=0}^{n-1} \frac{\sigma_2(i,m)}{\sigma_i(i+1,m)} + \mu_1(0,m)(1-\delta_{0m})G(n,m) - K(m)\mu_1(0,m+1)\,(1-\delta_{mM})G(n,m)\right], 1 \le n \le N \tag{27}$$

where $K(m)=\prod_{j-0}^{N-1}\frac{\{\sigma_2(j,m)\}/\{\sigma_1(j+1,m)\}+\mu_1(0,m)(1-\delta_{0m})G(N,m)}{\{\mu_1(0,m+1)\}/\{\mu_2(N,m)\}+\mu_1(0,m+1)G(N,m)}$

and $G(n,m)=1+\sum_{i=1}^{n-1}\prod_{j=1}^{i}\frac{\{\sigma_2(n-j,m)\}/\{\sigma_1(n-j,m)\}}{\sigma_1(n,m)}$.

$p_{0,0}$ can be determined by using normalizing condition.

7. STEADY STATE PROBABILITY VECTOR : MATRIX GEOMETRIC APPROACH

Consider two node-closed network with preemptive different priority disciplines at both nodes (left and right nodes). The steady state process under consideration can be formulated as a continuous time Markov Chain with state space $\{(n,m);\ 0\le n\le N,\ 0\le m\le M\}$ where *n* denotes high priority customers and *m* represents low priority customers.

The infinitesimal generator *Q* of the continuous time Markov chain is given by the tridiagonal matrix in which all the blocks may also be in matrix form as given below.

$$Q=\begin{bmatrix} B_{00} & A_0 & & & & \\ A_2 & A_1 & A_0 & & & \\ & A_2 & A_1 & A_0 & & \\ & & & \cdots\cdots & & \\ & & \cdots\cdots & & & \\ & & & A_2 & A_1 & A_0 \\ & & & & A_2 & A_1' \end{bmatrix} \tag{28}$$

where the matrices $B_{00}, A_0, A_1, A_2, A_1'$ are given as follows

$$B_{00}=\begin{bmatrix} \sigma_2+\mu_2 & \mu_1 & & & & \\ \mu_2 & \mu_2+\mu_1+\sigma_2 & \mu_1 & & & \\ 0 & \mu_2 & \mu_2+\mu_1+\sigma_2 & \mu_1 & & \\ & & \cdots\cdots & & & \\ & & & \cdots\cdots & & \\ & & & \mu_2 & \mu_2+\mu_1+\sigma_2 & \mu_1 \\ & & & & \mu_2 & \mu_1+\sigma_2 \end{bmatrix}_{(N+1)\times(N+1)}$$

$$\backslash A_0=\begin{bmatrix} \sigma_1 & & & & & \\ & \sigma_1 & & & & \\ & & \cdots\cdots & & & \\ & & & \cdots\cdots & & \\ & & & & \sigma_1 & \\ & & & & & \sigma_1 \end{bmatrix}_{(N+1)\times(N+1)}$$

$$A_1 = \begin{bmatrix} \mu_2+\sigma_1+\sigma_2 & & & & & \\ \mu_2 & \mu_1+\sigma_1+\sigma_2 & & & & \\ & & \cdots\cdots\cdots & & & \\ & & & \cdots\cdots\cdots & & \\ & & & \mu_2 & \mu_1+\sigma_1+\sigma_2 & \\ & & & & \mu_2 & \sigma_1+\sigma_2 \end{bmatrix}_{(N+1)\times(N+1)}$$

$$A_2 = \begin{bmatrix} \sigma_2 & & & & & \\ & \sigma_2 & & & & \\ & & \cdots\cdots\cdots & & & \\ & & & \cdots\cdots\cdots & & \\ & & & & \sigma_2 & \\ & & & & & \sigma_2 \end{bmatrix}_{(N+1)\times(N+1)}$$

$$A_1' = \begin{bmatrix} \mu_2+\sigma_1 & & & & & & \\ \mu_2 & \mu_2+\sigma_1 & & & & & \\ & \mu_2 & \mu_2+\sigma_1 & & & & \\ & & & \cdots\cdots\cdots & & & \\ & & & & \cdots\cdots\cdots & & \\ & & & & \mu_2 & \mu_2+\sigma_1 & \\ & & & & & \mu_2 & \sigma_1 \end{bmatrix}_{(N+1)\times(N+1)}$$

The steady state probability vector X, can be obtained by solving the system of equations XQ = 0 and the normalizing condition Xe =1 where e is a column vector of appropriate dimension having all elements equal to 1.
The finitesimal generator Q is irreducible and has a special block tridiagonal structure. Thus, the system of linear equations is solved by using the matrix geometric method given by Neuts (1981).
Let us partition the steady state probability vector X as

$$X = \left[X_0, X_1 X_2, \ldots\ldots\ldots, X_N\right] \tag{29}$$

where $X_0 = \left[X_{00}, X_{01} X_{02}, \ldots\ldots\ldots, X_{0,N}\right]_{1\times(N+1)}$ and $(X_i)_{1\times(N+1)}$ for $1 \le i \le N$.

By implementing matrix - geometric method followed by Neuts (1981), we may examine the existence of a solution of the form

$$X_i = X_1 R^{i-1} \quad , i \le 1 \tag{30}$$

or $X_i = X_{i-1} R \quad , \quad i \le 1$ (31)

The system of equation $XQ = 0$ is given by

$$X_0 B_{00} + X_1 A_2 = 0 \tag{32}$$

$$X_{i-1} A_0 + X_i A_1 + X_{i+1} A_2 = 0, \; i = 1,2,\ldots\ldots, N-1 \tag{33}$$

$$X_{N-1} A_0 + X_N A_1' = 0 \tag{34}$$

If there exists matrix geometric solution, then the system of equations

$XQ = 0$ given non-linear matrix equation

$$A_0 + RA_1 + R^2 A_2 = 0 \tag{35}$$

where R is a square matrix of order (N+1) and it is the unique minimal non-negative solution to the matrix equation (35) with R ≥ 0. It is an irreducible non-negative matrix of spectral radius less than one.

By successive substitution in the recurrence relation, the matrix R of order (N+1) can be computed as follows:

$$R(0) = 0 \tag{36}$$

$$R(n+1) = -A_0 A_1^{-1} - R^2(n) A_2 A_1^{-1} \text{ for } n \geq 0 \tag{37}$$

Now, we have to calculate the steady state probability vector $X = [X_0, X_1, X_2, \ldots\ldots\ldots, X_N]$ that is also called the matrix geometric probability vector.

For this purpose, we obtain the matrix form of the balance equations for the boundary states given by equations (31) and (35).

$$(X_0, X_1)\begin{bmatrix} B_{00} & A_2 \\ A_0 & A_1 + RA_2 \end{bmatrix} = 0 \tag{38}$$

where $X_2 = X_1 R$. Equation (38) does not have any unique solution.
The normalizing condition $Xe = 1$ gives

$$X_0 e + X_1\left[\left(\frac{1 - R^{N-1}}{1 - R}\right)e - A_0 (A_1')^{-1} R^{N-2} e\right] = 1 \tag{39}$$

which gives a unique solution for

$$X = [X_0, X_1, X_2, \ldots\ldots\ldots\ldots, X_N] \tag{40}$$

where e is a column matrix of appropriate dimension having all elements 1.

8. DISCUSSION

In this paper, we have analyzed closed two node queueing network with preemptive different priority discipline. The equilibrium distribution has been provided for closed two node queueing networks with preemptive repeat identical and preemptive resume discipline. The steady state probability vector derived for the queueing network by using matrix-geometric approach can also be employed to determine various system characteristics. The priority queueing models are encountered in the design of computer networks (ATM networks), manufacturing systems and transportation networks. The queueing model with preemptive different priority service discipline has the property that the service time of the preempted unit on its re-entry does not depend on past preemptions, so the cost of the waiting times and the cost of the interrupted service may be minimized. Thus, the time-sharing computer systems with preemptive repeat different priority provide faster responses and minimize delay times.

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