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FAULT TOLERANT SYSTEM WITH WORKING BREAK DOWN, IMPERFECT REPAIR AND SERVER VACATION

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ABSTRACT

The redundant fault tolerant system can be found in many industrial as well as in commercial organizations which operate in intelligent machining environment. The present investigation is focused on the performance prediction of a automated fault tolerant machining system in which the server is unreliable as such subject to unpredictable breakdown. The server may also breakdown during working vacation. The repair job of the server may be imperfect irrespective of its failure in normal busy mode or in working vacation mode. The server is allowed to go for a working vacation in case when there is no workload. The transient analysis of the automated fault tolerant machining system has been suggested by developing finite population queueing model and framing Chapman Kolmogorov equations. Numerical results using Runge-Kutta approach have been computed for various queueing and reliability indices established in terms of transient probabilities of the system states. The sensitivity of system descriptors with reference to performance metrics has also been examined.

Keywords: Fault tolerance, intelligent system, Automated recovery, Imperfect repair, Working vacation, Unreliable server, Reliability.

1. INTRODUCTION

The machining components embedded in the automated system are susceptible to failures which lead to system failure also. In fault tolerant machining systems, the provision of maintainability and redundancy can be done to cope up with the system breakdown due to both technical as well as non-technical faults. Sometimes, component failures lead to huge economic, time and energy losses. So around the globe, the system organizers and decision makers are working round the clock to counter such type of problems which are common in many real life situations such as ticket counters at railway, billing counters at mall, call centers, power plants, etc. The performance prediction by developing queueing models can play important role in order to make the cost efficient fault tolerance system and may be helpful to improve its reliability and throughput. In this investigation we are concerned with the stochastic modeling and performance analysis of unreliable machining system having the facility of repair and redundant support. The provision of working vacation and working breakdown is taken into account. By developing finite population queueing model, transient performance indices are established in terms of system state probabilities. For computation of queue size distribution of failed machines in the system, numerical approach based on Ruge-Kutta method is followed.

2. LITERATURE REVIEW

In the recent past, the industrial engineers and system analysts have put in too much effort in the field of automated fault tolerance machining system. Several queue theorists and system designers have developed the performance models of repairable redundant machining system. In order to tackle with the occurrence of failures and redundancy in a machining system, the researchers have to incorporate the standby support system and repair facility. Many prominent machine repair models have been proposed in the recent past by renowned researchers including Pósafalvi et al. (1987), Sivazlian and Wang (1989), Gupta (1997), Jain. (1997) and many others. A machine repair model along with repair/replacement policy was investigated by Jiang et al. (2001). The finite population model for machine repair problem (MRP) with N-policy was investigated by Jain et al. (2004). A state dependent system with standbys and two types of failures was examined by Jain et al. (2008). Markovian models for machining systems with standby support were also studied by Ke et al. (2008), and Ke et al. (2012) by incorporating service interruptions and multiple vacations, respectively. Jain et al. (2016) investigated a time dependent MRP with two types of standbys and N-policy in which cost analysis was also done. Jain and Jain (2017) investigated a MRP with two types of spares and vacation. Kumari et al. (2020) investigated a MRP with vacation and controlled by F-policy.

In automated machining systems, it is quite obvious that the repair facility can also fail. The situation of workshop failures and its repairs has also drawn the attention of researchers working in the field of queueing and reliability theory. Nguyan et al. (1981), Dohi et al. (1997), and Love et al. (1998) established repair/replacement policies for the systems the facing problem of imperfect repair. Optimal repair policies for a machining system with imperfect repair was studied by Sheu et al. (2005). Hajeeh et al. (2009) studied a machining system with imperfect repair and studied its performance by measuring the steady-state probability. In the last decade, a few researchers have proposed the queueing models and performance analysis for the machining system with imperfect repair. Nodem et al. (2011) suggested maintenance and replacement policies for the machining system which may be unavailable due to imperfect repairs. Pandey et al. (2013) studied a binary system with imperfect repair. Wu et al. (2019) scheduled maintenance policy for a production/ manufacturing system by considering the imperfect repairs. A replacement model for the deteriorating system with imperfect repair was studied by Dong et al. (2019). Liu et al. (2020) investigated the performance metrics of machining system by developing queueing model with imperfect repair.

The fault tolerant machining systems (FTMS) can be commonly seen in many diverse fields and hence it drives the attention of researchers. In a FTMS with maintenance support, the server can also go for a working vacation which means that the server will continue to serve even during its vacation but with a different rate. A very few research papers have appeared on the performance models of such systems in different frame-works including Wang et al. (2009), Yang et al. (2010) and Jain and Jain (2010). In the current decade, an M/M/2 machine repair problem was studied by Krishnamoorthy (2012) in which one server goes for the working vacation. Yang et al. (2015) developed a cost minimization model for a machining system with working vacation. Jain et al. (2017) studied a machining system with working vacation in which admission was controlled via F- policy. Jain et al. (2019) investigated a finite Markovian machining system with working vacation using matrix method. Ketema (2020) analyzed a multiple working vacation machining system operating under triadic policy.

In queueing literature, the presence of unreliable server in a machine repair problem has played a critical role in the development of many models. Some of the important recent past works on machining system with unreliable server can be attributed to Chakravarthy et al. (2001), Ke (2006), Wang et al. (2007), Wang and Yang (2009) and Wu et al. (2010). Over the last decade, a few investigations have been done in this direction by renowned researchers. Ke et al. (2013) investigated a machining system with multi unreliable servers. Jain et al. (2014) analyzed the machine repair system with unreliable server controlled by F-policy. Ke et al. (2018) developed Markovian model for the MRP having realistic features of unreliable server and imperfect switch over of the standbys. M/M/1 model along with cost optimization for the machining system with unreliable repairman was investigated by Sethi et al. (2020). Shekhar et al. (2021) studied an unreliable server machine repair problem with vacation interruptions.

3. MODEL DESCRIPTION

Consider the finite population Markov model for the machining system which is comprised of M operating machines, S number of warm standbys and Y number of cold standbys. The operating machines on failure are replaced by the warm standbys and later on by cold standbys. The failure characteristic of the standbys is similar to that of the operating machines. There is a provision of a server which is unreliable as it can go on working vacation when there is no repair job in the queue. The failure of server can occur while working in normal mode or on working vacation. When the server breaks down, there is a provision of its repair which may be successful or unsuccessful. When it is unsuccessful then there is facility of secondary repair also (see figure 1). The broken down server or server on vacation also renders the repair job with degraded rate.

3.1 Assumptions

The assumptions made for mathematical formulation of Markov model for performance prediction of FTMS are as follows:

- The life time of the machines (warm standbys) is exponentially distributed with rate $\lambda(a)$. The repair of machines is done by unreliable server; the repair times of machines are exponentially distributed with rate μ, μ_1 and μ_2 when the server is in normal working mode, working vacation mode and breakdown states, respectively.
- The life time of the server is exponentially distributed with rate α and α_v while broken down during working breakdown mode and working vacation mode, respectively.
- The repair times of the server while broken down during normal busy and working vacation mode of server are exponentially distributed with rate β and β_v, respectively.
- The rate of going on vacation by the server is ε. The exponential distribution with mean $\frac{1}{\theta}$ is assumed for the working vacation period.
- The probability of successful repair of the server during normal busy mode (working vacation mode) is σ (σ_v). The probability of unsuccessful repair of the server during normal busy mode is $\sigma'(= 1-\sigma)$ and the probability of unsuccessful repair of the server during working vacation mode is σ'_v $(= 1-\sigma_v)$.
- The rate of secondary repair in case of unsuccessful repair of the server when it failed during working vacation (working breakdown) is $\beta_e(\beta_O)$.

Now we define the different states of the system $\varphi(t)$ as,

$$\varphi(t) = \begin{cases} 0, & \textit{Repair of server is unsuccessful when broken down during WV} \\ 1, & \textit{Server is in normal busy state} \\ 2, & \textit{Server is in WV state} \\ 3, & \textit{Server is under repair when broken down during WV state} \\ 4, & \textit{Server is in under repair being broken down during normal busy state} \\ & \textit{and can perform repair job with degraded rate} \\ 5, & \textit{Repair of server is unsuccessful when broken down during} \\ & \textit{normal busy state} \end{cases}$$

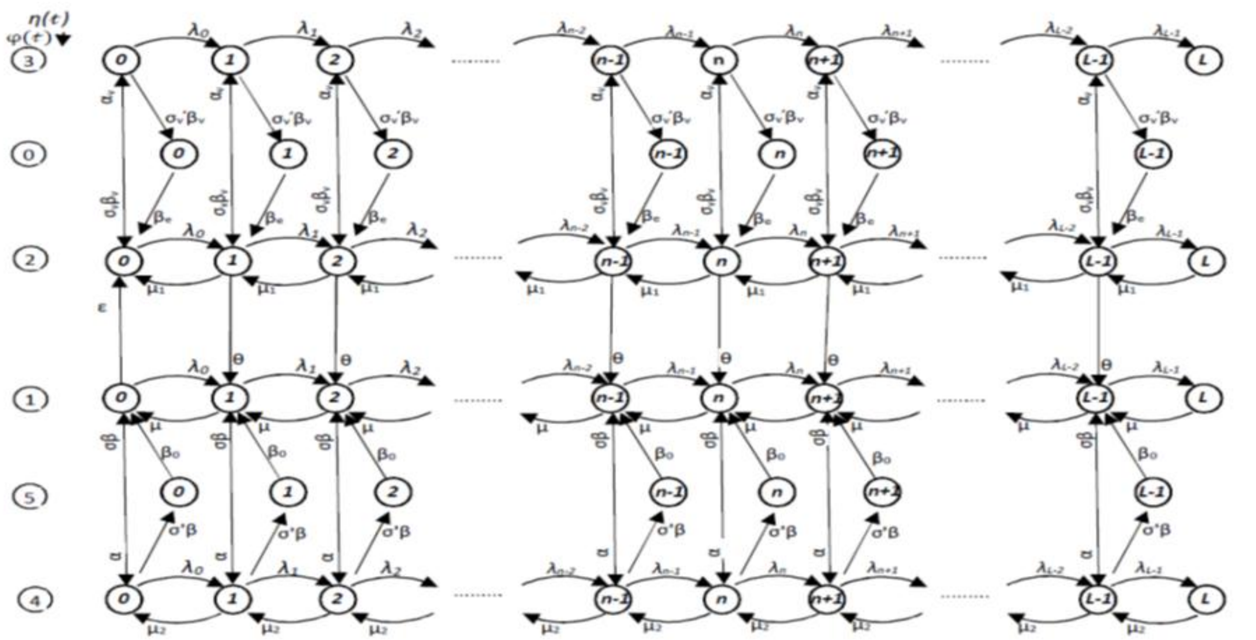

Figure 1: State Transition Diagram where $n(t)$ refers to the failed machines and $\varphi(t)$ refers to the states

The state dependent arrival rate (λ_i) of failed machines is defined as follow:

$$\lambda_i = \begin{cases} M\lambda + Sa; & 0 \le i \le S \\ M\lambda + (S+Y-i)a; & S < i \le S+Y \\ (M+S+Y-i+1)\lambda; & Y+S+1 \le i < L \end{cases}$$

Let $\mathcal{P}_{i,n}(t)$ denote the transient probabilities for different system states when $\varphi(t) = i$ and 'n' represents the number of failed units at time epoch 't'.

3.2 Governing Equations

Using the quasi birth-death process, we have constructed the following Chapman-Kolmogorov equations for the transient distribution of the number of failed machines in the system:

(i) Repair of the server is unsuccessful when broken down during working vacation ($\boldsymbol{\varphi(t) = 0}$).

$$\frac{d\mathcal{P}_{0,0}(t)}{d\tau} = \sigma_v{}'\beta_v\mathcal{P}_{3,0}(t) - \beta_e\mathcal{P}_{2,0}(t) \tag{1}$$
$$\frac{d\mathcal{P}_{0,n}(t)}{d\tau} = \sigma_v'\beta_v\mathcal{P}_{3,n}(t) - \beta_e\mathcal{P}_{2,n}(t); 0 < n < L \tag{2}$$

(ii) Server is in operating state ($\boldsymbol{\varphi(t) = 1}$).

$$\frac{d\mathcal{P}_{1,0}(t)}{d\tau} = -(\lambda_0 + \alpha + \varepsilon)\mathcal{P}_{1,0}(t) + \mu\mathcal{P}_{1,1}(t) + \beta_0\mathcal{P}_{5,0}(t) + \sigma\beta\mathcal{P}_{4,0}(t) \tag{3}$$
$$\frac{d\mathcal{P}_{1,n}(t)}{d\tau} = -(\lambda_n + \alpha + \mu)\mathcal{P}_{1,n}(t) + \lambda_{n-1}\mathcal{P}_{1,n-1}(t) + \theta\mathcal{P}_{2,n}(t) + \mu\mathcal{P}_{1,n+1}(t) + \beta_0\mathcal{P}_{5,n}(t) + \sigma\beta\mathcal{P}_{4,n}(t);\quad 0 < n < L \tag{4}$$
$$\frac{d\mathcal{P}_{1,L}(t)}{d\tau} = -\mu\mathcal{P}_{1,L}(t) + \lambda_{L-1}\mathcal{P}_{1,L-1}(t) \tag{5}$$

(iii) Server is in working vacation state ($\boldsymbol{\varphi(t) = 2}$).

$$\frac{d\mathcal{P}_{2,0}(t)}{d\tau} = -(\lambda_0 + \alpha_v)\mathcal{P}_{2,0}(t) + \varepsilon\mathcal{P}_{1,0}(t) + \mu_1\mathcal{P}_{2,1}(t) + \beta_v\mathcal{P}_{0,0}(t) + \sigma_v\beta_v\mathcal{P}_{3,0}(t) \tag{6}$$
$$\frac{d\mathcal{P}_{2,n}(t)}{d\tau} = -(\lambda_n + \alpha_v + \mu_v + \theta)\mathcal{P}_{2,n}(t) + \lambda_{n-1}\mathcal{P}_{2,n-1}(t) + \mu_1\mathcal{P}_{2,n+1}(t) + \beta_v\mathcal{P}_{0,n}(t) + \sigma_v\beta_v\mathcal{P}_{3,n}(t);\quad 0 < n < L \tag{7}$$
$$\frac{d\mathcal{P}_{2,L}(t)}{d\tau} = -\mu_1\mathcal{P}_{2,L}(t) + \lambda_{L-1}\mathcal{P}_{2,L-1}(t) \tag{8}$$

(iv) Server is broken down while failed during working vacation state ($\boldsymbol{\varphi(t) = 3}$).

$$\frac{d\mathcal{P}_{3,0}(t)}{d\tau} = -(\lambda_0 + \alpha_v\beta_v + \sigma_v{}'\beta_v)\mathcal{P}_{3,0}(t) + \alpha_v\mathcal{P}_{2,0}(t) \tag{9}$$

$$\frac{d\mathcal{P}_{3,n}(t)}{d\tau} = -(\lambda_n + \alpha_v\beta_v + \sigma_v{}'\beta_v)\mathcal{P}_{3,n}(t) + \lambda_{n-1}\mathcal{P}_{3,n-1}(t) + \alpha_v\mathcal{P}_{2,n}(t)\ ; 0 < n < L \tag{10}$$
$$\frac{d\mathcal{P}_{3,L}(t)}{d\tau} = \lambda_{L-1}\mathcal{P}_{3,L-1}(t) \tag{11}$$

(v) Server is under repair on being broken down and doing repair job with degraded rate ($\boldsymbol{\varphi(t) = 4}$).

$$\frac{d\mathcal{P}_{4,0}(t)}{d\tau} = -(\lambda_0 + \sigma\beta + \sigma'\beta)\mathcal{P}_{4,0}(t) + \alpha\mathcal{P}_{1,0}(t) + \mu_2\mathcal{P}_{4,1}(t) \tag{12}$$
$$\frac{d\mathcal{P}_{4,n}(t)}{d\tau} = -(\lambda_n + \sigma\beta + \sigma'\beta + \mu_d)\mathcal{P}_{4,n}(t) + \lambda_{n-1}\mathcal{P}_{4,n-1}(t) + \alpha\mathcal{P}_{1,n}(t) + \mu_2\mathcal{P}_{4,n+1}(t);\quad 0 < n < L \tag{13}$$
$$\frac{d\mathcal{P}_{4,L}(t)}{d\tau} = -\mu_2\mathcal{P}_{4,L}(t) + \lambda_{L-1}\mathcal{P}_{4,L-1}(t) \tag{14}$$

(vi) Repair of the server is unsuccessful when broken down during normal busy state ($\boldsymbol{\varphi(t) = 5}$).

$$\frac{d\mathcal{P}_{5,0}(t)}{d\tau} = \sigma'\beta\mathcal{P}_{4,0}(t) - \beta_o\mathcal{P}_{5,0}(t) \tag{15}$$

$$\frac{d\mathcal{P}_{5,n}(t)}{d\tau} = \sigma'\beta\mathcal{P}_{4,n}(t) - \beta_o\mathcal{P}_{1,n}(t); 0 < n < L \tag{16}$$

The system of differential equations (1)−(16) are difficult to solve analytically. We shall compute the transient state probabilities by employing Runge−Kutta method of fourth order to solve equation (1)−(16) along with initial condition $\mathcal{P}_{1,0}(0) = 1$; $\mathcal{P}_{i,n}(0) = 0$ $when$ $i \neq 1, n \neq 0$. The set of equations (1)-(16) can be written as:

$$\frac{d\boldsymbol{\mathcal{P}}(t)}{d\tau} = f\big(t, \mathcal{P}(t)\big) \tag{17}$$

where unknown probability vector

$$\boldsymbol{\mathcal{P}}(t) = \big[\mathcal{P}_{0,0}(t), \ldots \mathcal{P}_{0,L-1}(t), \mathcal{P}_{1,0}(t), \ldots \mathcal{P}_{1,L}(t), \mathcal{P}_{2,0}(t), \ldots, \mathcal{P}_{2,L}(t), \ldots, \mathcal{P}_{5,0}(t), \ldots, \mathcal{P}_{5,L-1}(t)\big], \tag{18}$$

can be computed by using routine ode45 of MATLAB 19.

4. PERFORMANCE INDICES

In order to endorse the efficacy of present model in real time system, we shall establish some measures of performance. Thus the following performance indices have been formulated in terms of transient state probabilities of machining system supported by standbys and repair facility.

(i) Expected number of failed machines in the system at time t

$$E_F(t) = \sum_{i=1}^{4}\sum_{n=1}^{L} n\mathcal{P}_{i,n}(t) \tag{19}$$

(ii) Probability of the server when it is working in normal mode at time t

$$\mathcal{P}_{NB}(t) = \sum_{n=0}^{L}\mathcal{P}_{1,n}(t) \tag{20}$$

(iii) Probability of the server state when it is in working vacation at time t

$$\mathcal{P}_{WVA}(t) = \sum_{n=0}^{L}\mathcal{P}_{2,n}(t) \tag{21}$$

(iv) Probability that the server is in breakdown at time t

$$\mathcal{P}_B(t) = \sum_{n=0}^{L}\mathcal{P}_{4,n}(t) \tag{22}$$

(v) Availability of machine at time t

$$\mathcal{A}v(t) = 1 - \frac{E_F(t)}{M+Y+S+1} \tag{23}$$

(vi) Reliability of the system at time t

$$\mathcal{R}_L(t) = 1 - \sum_{i=1}^{4}\mathcal{P}_{i,L}(t) \tag{24}$$

(vii) Throughput of the system at time t

$$\mathcal{T}\hbar(t) = \sum_{n=1}^{L}\mu\mathcal{P}_{1,n}(t) + \sum_{n=1}^{L}\mu_1\mathcal{P}_{2,n}(t) + \sum_{n=1}^{L}\mu_2\mathcal{P}_{4,n}(t) \tag{25}$$

(viii) Carried load in the system at time t

$$\mathcal{L}(t) = \sum_{i=1}^{4}\sum_{n=0}^{L-1}\lambda_n\mathcal{P}_{i,n}(t) \tag{26}$$

4.1 Cost function

The cost function for the FTMS has been constructed to make the system economic by the optimal choice of repair rates. We intend to minimize the cost by setting the optimal service rates. To construct the cost function, we have considered the different cost factors which are as follows:

$\mathcal{C}_O$ = Cost per unit time of the failed machines when the system is working in normal mode.
$\mathcal{C}_N$ = Cost per unit time when the server is in normal mode.
$\mathcal{C}_B$ = Cost per unit time when the server is in working breakdown state.
$\mathcal{C}_V$ = Cost per unit time when the server is in working vacation.
$\mathcal{C}_{Sb}$ = Cost per unit time of the secondary repair of the server during working breakdown.
$\mathcal{C}_{Sv}$ = Cost per unit time of the secondary repair of the server during working vacation.
$\mathcal{C}_3$= Cost per unit time of the primary repair of the server during working breakdown.
$\mathcal{C}_4$= Cost per unit time of the primary repair of the server during working vacation.
$\mathcal{C}_N$ = Cost per unit time of the repair done by the repairman at normal rate.
$\mathcal{C}_1$= Cost per unit time of the repair done by the repairman during working vacation.

$\mathcal{C}_2$= Cost per unit time of the repair done by the repairman during breakdown.

The cost parameters are assumed to be linear and formulated as follows:

$$\mathcal{C}ost(t) = \mathcal{C}_O E_F(t) + \mathcal{C}_N \mathcal{P}_{NB}(t) + \mathcal{C}_B \mathcal{P}_B(t) + \mathcal{C}_V \mathcal{P}_{WVA}(t) + \beta_0 \mathcal{C}_{Sb} + \beta_e \mathcal{C}_{Sv} + \beta \mathcal{C}_3 + \beta_v \mathcal{C}_4 + \mu \mathcal{C}_N + \mu_1 \mathcal{C}_1 + \mu_2 \mathcal{C}_2 \tag{27}$$

5. NUMERICAL ANALYSIS

We employ Runge-Kutta method for the computation of performance metrics for FTMS by taking an illustration of repair facility available to perform repair jobs of the broken down terminals of the computer center in an academic institute. In computer center, there is facility of a finite number of terminals (say L) along with some mixed type of standby terminals. For the smooth functioning of the center, a skilled operater has been deputed which refurbish the failed terminal immediately. But the repairman is unreliable as it can also breakdown as well as can go on working vacation. For the smooth functioning of the computer center, the provision of the repair of the operator is also made. If the repair of the operator is unsuccessful then its secondary repair is also available so that the operator can be made available to perform repair job of terminals in normal mode.

The performance indices established in the previous section are used to compute these measures by taking default data compatible for the above mentioned illustration. By keeping default parameters fixed as $M = 4,\ S = 2,\ Y = 1,$ $a = 1$, $\alpha = 0.2$, $\alpha_v = 0.2$, $\beta_v = 4$, $\mu_v = 1, and\ \varepsilon = 0.3$ and some of the other parameters values, the numerical results obtained are displayed in tables 1-5 and figures 2-3. The trends of performance metrics with respect to system descriptors of the FTMS can be interpreted as follows.

5.1 Effect of time (t)

The expected number of failed machines *E_f (t)* in the system and the throughput of the system *Th(t)* increase (see tables 1-5 and figures 3(i)-3(iv)) as time *t* increases. On the contrary, the machine availability *MA(t),* t carried load *L(t)* and reliability of the system *R(t)* decrease (see tables 1-5 and figures 2(i)-2(iv)) with the increase in time time *t* as expected in the real time automated machining system too.

5.2 Effect of θ

The expected number of failed machines *E_f(t)* decreases with the rise in the value of θ as shown in table 1. As observed in the figure 3(iii), with the increasing values of θ, the throughput of the system *Th(t)* first remains almost unaffected but later on increases. On the other hand, the reliability of the system *R(t)*, the machine availability *MA(t)* and the carried load of the system *L(t)* increase on increasing the values of θ as elucidated in the figure 2(iii) and table 1.

5.3 Effect of repair rates (μ & μ_1) and the failure rate (λ)

At any particular time t, the increase in the repair rate of the repairmen $\mu(\mu_1)$ results in the decrement in the expected number of failed machines in the system *E_f(t)* whereas the machine availability *MA(t)* and the carried load capacity of the system *L(t)* increase with the increase in $\mu(\mu_1)$ as displayed in tables 2(3).

With the increase in the failure rate λ of the operating machines, the expected number of failed machines *E_f(t)* increases whereas the machine availability *MA(t)* and the carried load of the system *L(t)* decrease which is quite obvious and also noticed from table 4. The reliability *R(t)* and throughput *Th(t)* of the system decrease with the increase in the failure rate λ as clear from figure 2(i) and figure 3(iii), respectively. The reliability of the system *R(t)* increases with the increase in μ as depicted in figure 2(ii). The throughput of the system *Th(t)* increases with the increase in the value of μ_1 which can be noticed in the figures 3(ii) which is obvious too.

5.4 Effect of the repair rate β(β_v) and secondary repair rate β_0 (β_e) during breakdown (vacation).

The expected number of failed machines *E_f(t)*, the machine availability *MA(t)* and the carried load of the system *L(t)* increase slightly with the increase in β_0 as depicted in the table 5. The reliability of the system *R(t)* remains constant with the increase in β as shown in figures 2(iv). With the increase in the β_e, the throughput of the system *Th(t)* increases as shown in figure 3(iii).

Based on the trends of numerical results obtained we conclude that the system will be more reliable if repair rates is higher which matches with the experience on real time system, If the failure rate of the machines is more, then the system will be less available. In order to cope up with this situation, it is advisable to adopt the optimal combination of maintainability and redundancy.

Table 1: $E_f(t)$, *MA(t)*, P_{NB} *(t)*, P_{WV}*(t)* and *L(t)* for varying θ

	T	$E_f(t)$	MA(t)	$P_{NB}(t)$	$P_{WV}(t)$	L(t)
$\theta = 1$	2	6.8400	0.1450	0.3240	0.4369	4.4902
	4	6.9884	0.1265	0.3708	0.3935	4.1108
	6	7.0680	0.1165	0.4093	0.3510	3.8949
$\theta = 2$	2	6.8062	0.1492	0.4912	0.2668	4.6539
	4	6.9516	0.1310	0.5414	0.2190	4.2974
	6	7.0301	0.1212	0.5776	0.1780	4.0951
$\theta = 3$	2	6.7858	0.1518	0.5819	0.1740	4.7540
	4	6.9315	0.1336	0.6253	0.1326	4.4023
	6	7.0111	0.1236	0.6528	0.1000	4.1988

Table 2 : $E_f(t)$, *MA(t)*, P_{NB} *(t)*, P_{WV}*(t)* and *L(t)* for varying μ

	t	$E_f(t)$	MA(t)	$P_{NB}(t)$	$P_{WV}(t)$	L(t)
$\mu = 5$	2	6.8400	0.1450	0.3240	0.4369	4.4902
	4	6.9884	0.1265	0.3708	0.3935	4.1108
	6	7.0680	0.1165	0.4093	0.3510	3.8949
$\mu = 6$	2	6.7777	0.1528	0.3233	0.4370	4.7343
	4	6.9134	0.1358	0.3697	0.3936	4.4158
	6	6.9827	0.1272	0.4077	0.3511	4.2510
$\mu = 7$	2	6.7141	0.1607	0.3228	0.4370	4.9718
	4	6.8366	0.1454	0.3688	0.3937	4.7138
	6	6.8950	0.1381	0.4063	0.3512	4.5999

Table 3 : $E_f(t)$, *MA(t)*, P_{NB} *(t)*, P_{WV}*(t)* and *L(t)* for varying μ_v

	t	$E_f(t)$	MA(t)	$P_{NB}(t)$	$P_{WV}(t)$	L(t)
$\mu_v = 4$	2	6.8400	0.1450	0.3240	0.4369	4.4902
	4	6.9884	0.1265	0.3708	0.3935	4.1108
	6	7.0680	0.1165	0.4093	0.3510	3.8949
$\mu_v = 5$	2	6.7207	0.1599	0.3318	0.4277	4.9524
	4	6.8795	0.1401	0.3823	0.3800	4.5514
	6	6.9711	0.1286	0.4239	0.3338	4.2979
$\mu_v = 6$	2	6.5995	0.1751	0.3375	0.4209	5.4011
	4	6.7699	0.1538	0.3908	0.3700	4.9768
	6	6.8754	0.1406	0.4348	0.3212	4.6821

Table 4 : $E_f(t)$, *MA(t)*, $P_{NB}(t)$, $P_{WV}(t)$ and *L(t)* for varying λ

	t	$E_f(t)$	MA(t)	$P_{NB}(t)$	$P_{WV}(t)$	L(t)
$\lambda = 4$	2	6.8400	0.1450	0.3240	0.4369	4.4902
	4	6.9884	0.1265	0.3708	0.3935	4.1108
	6	7.0680	0.1165	0.4093	0.3510	3.8949
$\lambda = 5$	2	7.0894	0.1138	0.3025	0.4277	3.9826
	4	7.1960	0.1005	0.3442	0.3887	3.6943
	6	7.2505	0.0937	0.3794	0.3504	3.5458
$\lambda = 6$	2	7.3697	0.0788	0.4152	0.4152	3.4610
	4	7.4339	0.0708	0.3814	0.3814	3.2708
	6	7.4646	0.0669	0.3477	0.3477	3.1839

Table 5 : $E_f(t)$, *MA(t)*, $P_{NB}(t)$, $P_{WV}(t)$ and *L(t)* for varying β_0

	t	$E_f(t)$	MA(t)	$P_{NB}(t)$	$P_{WV}(t)$	L(t)
$\beta_0 = 3$	2	6.8400	0.1450	0.3240	0.4369	4.4902
	4	6.9884	0.1265	0.3708	0.3935	4.1108
	6	7.0680	0.1165	0.4093	0.3510	3.8949
$\beta_0 = 4$	2	6.8400	0.1450	0.3240	0.4369	4.4923
	4	6.9884	0.1265	0.3708	0.3935	4.1136
	6	7.0680	0.1165	0.4093	0.3510	3.8986
$\beta_0 = 5$	2	6.8383	0.1452	0.3243	0.4369	4.4938
	4	6.9856	0.1268	0.3713	0.3935	4.1157
	6	7.0640	0.1170	0.4100	0.3510	3.9011

6. CONCLUSION

The prediction of the performance measures enables the system administrator to intelligently control the threshold parameters according to pre-specified requirement of availability of automated machining system which may operate under certain techno-economic constraints due to unavoidable faults/failures. The investigation done on the performance analysis of automated FTMS will be helpful in developing an efficient and reliable system which is fortified with warm and cold type of standbys and secondary repair facility to obtain a flawless service and has wide applications in various fields namely production systems, call centers, distributed computing centers, shopping outlets, distributed computer network, telecommunication systems, and many more.

Off course there are some limitations too as the working vacation and working breakdown can lead to higher costs and trigger on subsequent more faults if system works for longer duration in such situation. This investigation done can be modified by incorporating degraded/load sharing failure rates, switching failure of standby machines, multi-servers, etc. There is also scope of study on threshold based control policy for the repair and using general distributions for life time and/repair time for component failures in intelligent machining systems.

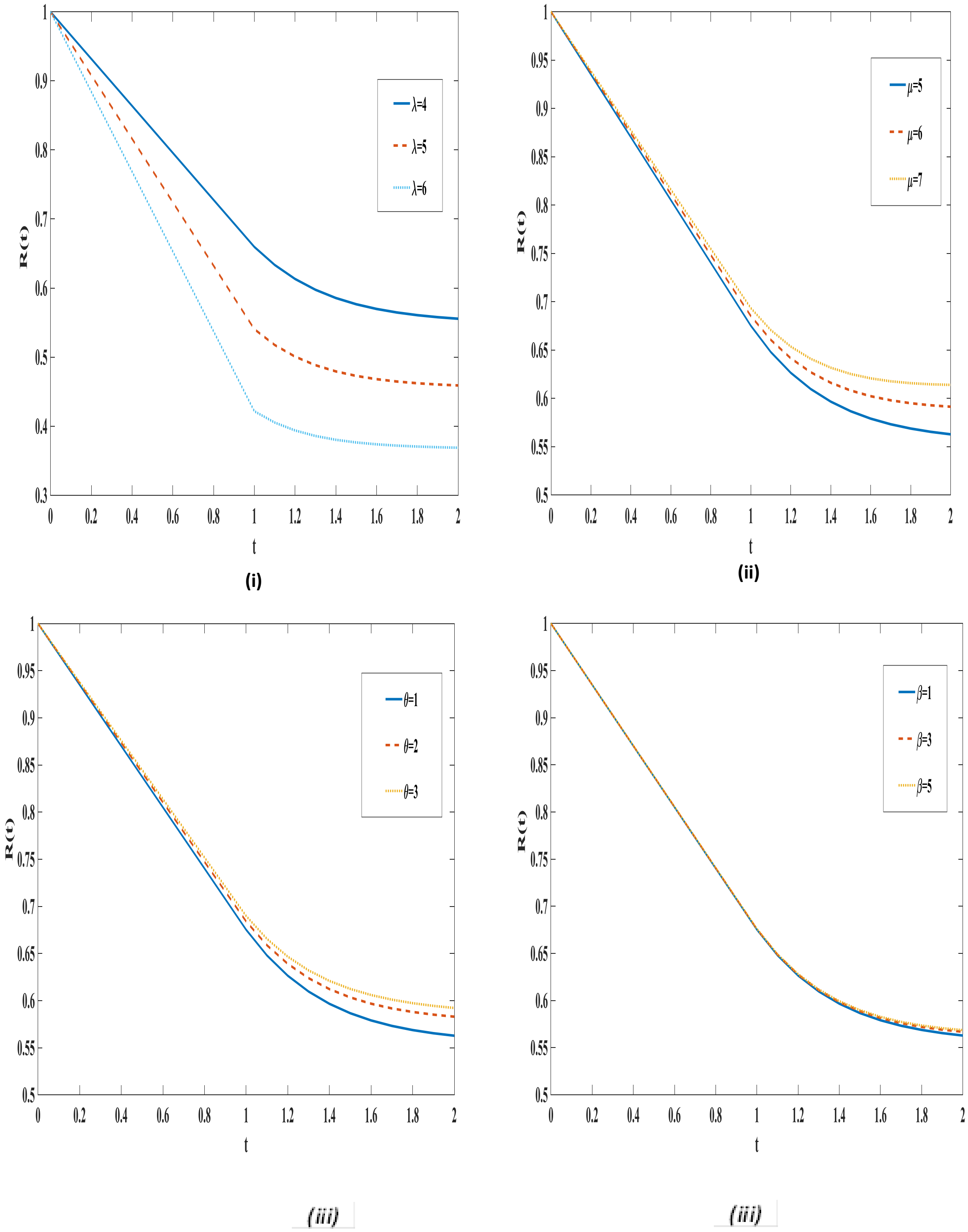

Figure 2: Reliability w.r.t. λ, μ, θ and β

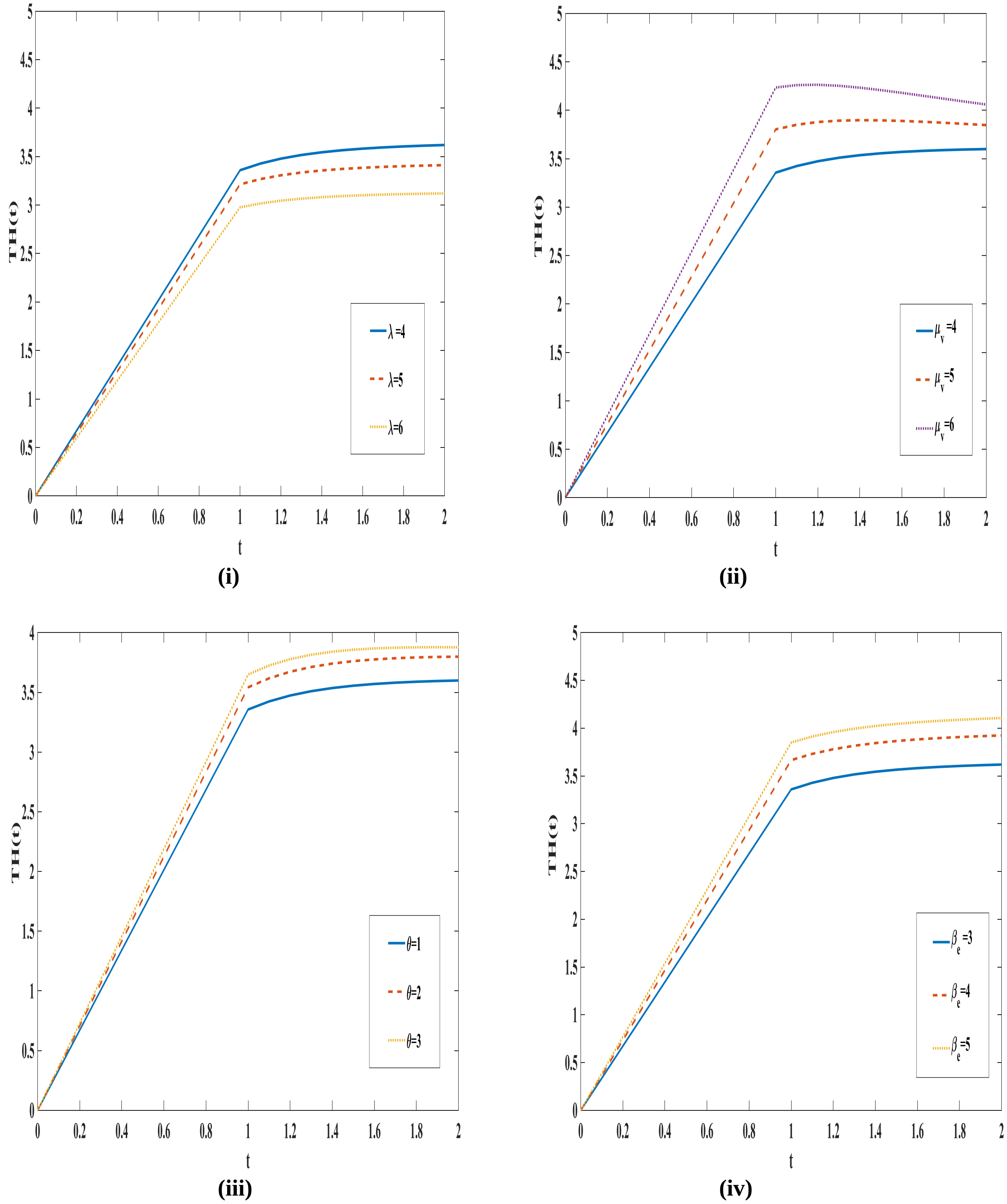

Figure 3: Throughput of the system w.r.t. λ , μ_v, θ and β_e

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