Global Journal of Modeling and Intelligent Computing (GJMIC)

Vol. 2, No. 2, July, 2022 ISSN: 2767-1917

CONTINUOUS SERVICE QUEUE WITH DISCOURAGEMENT AND IMAGINARY CUSTOMERS

Shobha Rani, Indian Institute of Technology Roorkee, India, (srani1@ma.iitr.ac.in)
Madhu Jain, Indian Institute of Technology Roorkee, India, (madhu.jain@ma.iitr.ac.in)
Nidhi Sharma, Indian Institute of Technology Roorkee, India, (n_sharma@ma.iitr.ac.in)

ABSTRACT

This article is concerned with the performance analysis of M/M/1 queue having two types of the customer's namely real and imaginary customers. The imaginary customers are served by the server in case if there is no real customer. Real (imaginary) customers are treated as regular (virtual) customers in the service system who join the system according to Poisson process. The priority is given to the real customers in comparison to imaginary customers. The real customer doesn't try to avert the service of ongoing imaginary customer. The recursive method is implemented to determine the steady state probabilities and other performance measures of the system. The numerical simulation is carried out to examine the effects of system descriptors on indices. Furthermore, ANFIS is used to facilitate the numerical results which are compared with the analytical results.

Keywords: Continuous Service, Markovian queue, Discouragement, Imaginary Customers, Queue length.

1. INTRODUCTION

Some researchers have shown an interest in resolving the queueing problems related to real-world scenarios having real and imaginary customers. There are numerous ways of analyzing the system better to understand the queueing characteristics and performance indices. In continuous service time queueing systems, the customers can be categorized into two groups i.e. real and imaginary customers. In call centers, the system has a large number of virtual/online callers (imaginary customers) waiting for their turn for service and real customers; however real customers are always treated with priority. Many real-world applications can be found in congestion scenarios wherein both real and imaginary customers join the system including call centers, cloud computing systems, medical and healthcare facilities, etc.

It is observed that most of the existing queueing models facilitate analysis of real customers in variant queueing situations. A very few researchers have shown their interest to solve queueing problems with continuous service having a single server and two type of customer's i.e., real, and imaginary customers. Some studies have also been done on the queueing systems with emergence of negative customers (Harrison & Pitel, 1996; Yang et al., 2001). They derived the queue length distribution in the closed-form. Panda & Goswami (2021) presented an economic analysis of a single vacationing server queue with the concept of positive and negative customers. A few researchers have studied the finite queueing models with discouragement where service is performed even in the absence of a customer or through phone calls. Kumar et al., (2021) investigated the single server queue in health care queue management system and discussed the impatience behavior of the customers. They evaluated the transient probabilities with the help of Runge-Kutta method. Jain et al. (2022) considered the Markovian vacation queue by involving the balking behavior of the customers and imperfect service. They derived the queue distribution by using the probability generating function.

Due to the complexity of the evaluation of queue size distribution and other performance measures via analytical approaches, many researchers have used the soft computing techniques. In this direction adaptive neuro-fuzzy inference system (ANFIS), which is a combination of fuzzy inference system (FIS) and artificial neural network (ANN) can play a vital role (Lin & Liu, 2003; Yang & Zhao, 2012). For the traffic signal controller at the traffic intersection, ANFIS approach is used to study the M/M/1 queueing model and its performance indices such as in terms of average waiting time and average queue size (Lai et al., 2015; Soh et al., 2011). Santana et al. (2019)

trained an ANFIS network to calculate the average time for all tasks for the scheduling purpose by developing of the standard queueing model based on uncertain data. Sanga & Jain (2019) investigated the finite Markovian queueing model with general retrial times and customers' discouraging behavior. They implemented the recursive formulae and ANFIS computation to establish several performance indices.

Our proposed model is fitted to many real-life scenarios. To illustrate we refer a health care management system. In hospital, there may be two types of patients (customers): (i) traditional patient (real customer) who receive the health care through face-to-face interaction with doctor and (ii) virtual patient (imaginary customer) to whom doctor can provide online or telephonically treatment by a method of streamlining care, potentially saving the time and expenses of physical visits. In hospital, a doctor cannot engage in virtual consultation while providing services to a traditional patient. When doctor observes that there are no more traditional patients in the system, the doctor connects to the virtual patient. If a traditional patient arrives during the service of a virtual patient and notices that the doctor is busy, he may exit (balk). During completion of virtual patient services, if another traditional patient arrives, then the doctor will provide the service to him/her. The expansion of electronic communication and electronic health records (EHRs) provide alternative means for the patient and physicians to interact. Virtual consultation has redefined healthcare as an effective way to alleviate the ongoing strain on hospitals battling with the Coronavirus pandemic. Tele-medicine offers a safe digital consultation to the patients at the comfort of their homes through a secure video or phone format.

The consideration of continuous service times is a novel idea for the queueing model. The main distinction between a regular M/M/1 queueing system and a continuous service time M/M/1 queue is that in the continuous service time M/M/1 queue, service will never stop, i.e., the server will never be idle in the system. If the server is in idle mode, then imaginary customers will appear, and the server will serve them. The server gives precedence to the real customers over the imaginary customers. For infinite and finite number of real customers in the system, we develop two Markovian models with discouragement and imaginary customers. The service times for both customers are exponentially and identically distributed. In order to get the results for the queue size distribution at steady state, we formulate the Markovian model with imaginary customers by using the birth death process. To derive several key performance indices, the steady-state probabilities have been established in closed form. The rest of the chapter is organized as follows. The model description with the suitable notations is given in section 6.2. The steady-state distributions for the infinite and finite capacity models have been derived in section 6.3. Some performance indices are established along with the waiting time distribution in in section 6.4. Section 6.5 is devoted to the real-life justification of the models. The numerical simulation and ANFIS results are provided in section 6.6.

2. MODEL DESCRIPTION

Markovian model with continuous service time queue is considered to find out the steady state probabilities. The real and imaginary customers join the system in Poisson fashion and can wait for the service. The proposed Markovian queueing model with continuous service time is quite close to standard $M / M /1$ queue while framing the governing equations. The counting of real (imaginary) customers is denoted by the random variables having positive (negative) integral values. We develop birth-death model of the continuous time system by using the arrival and service rates. When the server is dealing for the real customers then the system is in positive state. If imaginary customers are served by the server, then the system state is considered to be negative. The random variable M denotes the system size and takes both positive and negative values as follows:

- $M = 0$, refers that there is no real customers in the system and the server is attending the imaginary customers.
- When M takes the value 1, it implies that there is one real customer at the service counter and no real customers in the queue.
- $M = -1$ represents the system state when there is one real customer in the queue and the server is attending imaginary customer.
- $M = 2$ denotes the system state when it has 2 real customers with 1 real customer is standing in a queue and other one is receiving the service.
- $M = -2$ represents the state when system has 2 real customers in the queue and one imaginary customer in the service.

- If $M = K$, it implies that system has K real customers; $K - 1$ customers are waiting in a queue while one customer is currently receiving the service at the service counter.
- When $M = -K$, then there are K customers in the system who are waiting in the queue while the service provider is currently performing the service to the imaginary customer.

Now the steady state probability is defined by $P_K = \text{prob.}\ (M = K)$ i.e., probability of the system being in Kth state, where $K = \ldots -3, -2, -1, 0, 1, 2, 3\ldots$ The following assumptions and notations are used to develop the continuous service Markovian model.

- The real customers and imaginary customers arrive in the system following Poisson rule with rate λ and λ_I, respectively.
- The customer joins the queue with probability q in the positive state and may balk with complimentary probability $\bar{q} = 1 - q$.
- The customers (real and imaginary) are served by following the exponential distribution with rate $\mu(\mu_I)$.
- We denote $\rho = \lambda q / \mu$, $\rho_I = \lambda_I / \mu$ and $\rho_{I_1} = \lambda_I / \mu_I$.

3. THE ANALYSIS

Using birth death process, the governing equations have been derived for both the infinite and finite capacity Markov models with continuous service and balking.

3.1 Continuous service M/M/1/∞ model

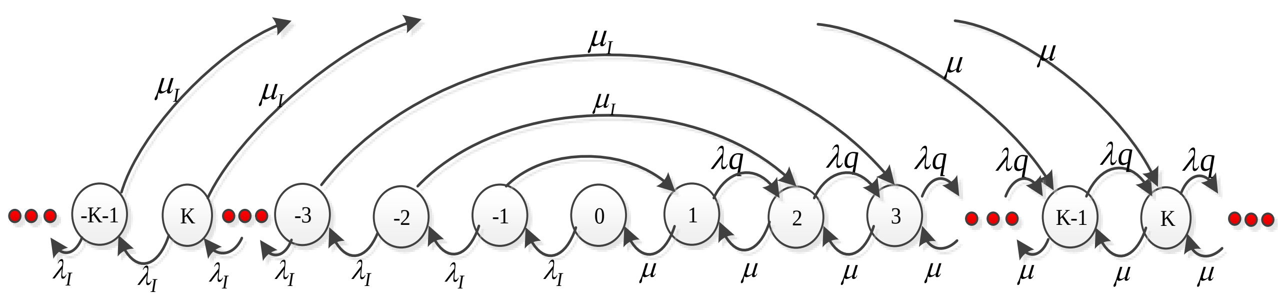

Figure 1: Transition diagram for M/M/1 queue with imaginary customers.

To obtain the steady state probabilities of K real customers in the system (P_K), where $\text{K} = \ldots -3, -2, -1, 0, 1, 2, 3\ldots$, we construct the balance equations as follows:

i. For imaginary customers:

- *When system state lies between* 0 *to* -1

$$\lambda_I p_0 = \mu_I (p_{-1} + p_{-2} + p_{-3} + \ldots) \tag{1}$$

- *When system state lies between -K+1 to -K*

$$\lambda_I p_{-K} = \mu_I (p_{-K-1} + p_{-K-2} + p_{-K-3} + \ldots) \tag{2}$$

ii. For real customers:

- *When system state lies between* 0 *to* 1

$$\mu p_1 = \mu_I (p_{-1} + p_{-2} + p_{-3} + \ldots) \tag{3}$$

- *When system state lies between* 0 *to* -1

$$\mu p_{K+1} = \lambda q p_{K-1} + \mu_I (p_{-K} + \mu p_{-(K+1)} + \ldots) \tag{4}$$

Steady state solution:

Solving eqs. (1)-(2) and (3)-(4), we get

$$\lambda_I p_0 = \mu p_1 \tag{5}$$

and $$\lambda_I p_{-K+1} = \mu p_K - \lambda q p_{K-1}, \quad K = 2,3,... \tag{6}$$

From eqs. (6.5)-(6.6), we get

$$(p_1 + p_2 + p_3 + ...) = \rho_I (p_0 + p_{-1} + p_{-2} + p_{-3} + ...) + \rho(p_1 + p_2 + p_3 + ...) \tag{7}$$

Dividing eq. (6.1) by λ_I , we get

$$p_0 = \frac{1}{\rho_{I_1}}(p_{-1} + p_{-2} + p_{-3} + ...) \tag{8}$$

From eqs. (6.2) & (6.8), we can obtain

$$p_{-K} = \left(\frac{\rho_{I_1}}{1+\rho_{I_1}}\right)^K p_0 \tag{9}$$

By using the normalizing condition, we have

$$p_0 + p_1 + p_{-1} + p_2 + p_{-2} + p_3 + p_{-3} + ... = 1 \tag{10}$$

Eq. (6.7) can be rewrite as

$$(p_1 + p_2 + p_3 + ...) = \left(\frac{\rho_I}{1+\rho}\right)(p_0 + p_{-1} + p_{-2} + p_{-3} + ...) \tag{11}$$

By using eq. (6.10) in above expression (11), we have

$$(p_1 + p_2 + p_3 + ...) = \frac{\rho_I}{(1-\rho+\rho_I)} \tag{12}$$

By using eqs. (12) into (10), we obtain

$$(p_0 + p_{-1} + p_{-2} + p_{-3} + ...) = \frac{1-\rho}{(1-\rho+\rho_I)}. \tag{13}$$

Using eqs. (6.9) into (6.13), we obtain the value of p_0 as

$$p_0 = \frac{1-\rho}{(1-\rho+\rho_I)(1+\rho_{I_1})}. \tag{14}$$

Dividing eqs. (6.3) - (6.5) by μ , we get the value of p_K as

$$p_1 = \frac{\mu_I}{\mu}(p_{-1} + p_{-2} + p_{-3} + ...)$$

$$p_{K+1} = \frac{\mu_I}{\mu}\left(p_{-(K+1)} + p_{-(K+2)} + ...\right) + \rho p_K. \tag{15}$$

By using eq. (6.8) in (6.15), we get

$$p_1 = \frac{\rho_I(1-\rho)}{(1-\rho+\rho_I)(1+\rho_{I_1})} \tag{16}$$

We obtain the value of p_2 with the help of eqs. (6.16) and (6.2) as

$$p_2 = \frac{\rho_I(1-\rho)}{(1-\rho+\rho_I)(1+\rho_{I_1})}A_{I_1} \tag{17}$$

where $A_{I_1} = \left[\frac{\rho_I}{(1+\rho_I)} + \rho\right]$.

Using eqs. (6.16), (6.17) and (6.2), we obtain p_K as

$$p_K = \frac{\rho_I(1-\rho)}{(1-\rho+\rho_I)(1+\rho_{I_1})}A_{I_{K-1}} \tag{18}$$

where $A_{I_{K-1}} = \left(\frac{\rho_I}{(1+\rho_I}\right)^{K-1} + \rho A_{K-2}$.

Now we obtain the steady state queue size distribution by collecting all the probabilities as

$$P_n = \begin{cases} \left(\frac{\rho_{I_1}}{1+\rho_{I_1}}\right)^n \frac{1-\rho}{(1-\rho+\rho_I)(1+\rho_{I_1})}; & n=-1,-2,-3... \\ \frac{1-\rho}{(1-\rho+\rho_I)(1+\rho_{I_1})} & n=0 \\ \frac{\rho_I(1-\rho)}{(1-\rho+\rho_I)(1+\rho_{I_1})}A_{I_{n-1}}; & n=1,2,3,... \end{cases} \tag{19}$$

where $A_{I_{n-1}} = \left(\frac{\rho_I}{(1+\rho_I}\right)^{n-1} + \rho A_{I_n}$, and $A_0 = 1$.

(i) For imaginary customers:

- *When system state lies between* 0 *to* -1

$$\lambda_I p_0 = \mu_I(p_{-1} + p_{-2} + p_{-3} + ... + p_{-N}) \tag{20}$$

6.3.2 Continuous service M/M/1/N model

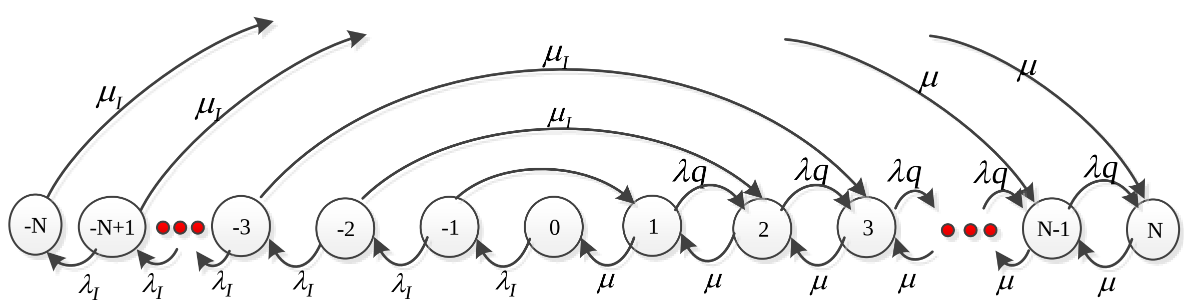

Figure 2: Transition diagram for M/M/1/N queue with imaginary customers.

- *When system state lies between -K to -K-1, K=2,3…*

$$\lambda_I p_{-K+1} = \mu_I(p_{-K-1} + p_{-K-2} + ... + p_{-N}) \tag{21}$$

- *When system state lies between* $-N+1$ *to* $-N$

$$\lambda_I p_{-N+1} = \mu_I(p_{-N}) \tag{22}$$

(ii) ***For real customers*:**

- *When system state lies between* 0 *to* 1

$$\mu p_1 = \mu_I (p_{-1} + p_{-2} + p_{-3} + \ldots + p_{-K}) \tag{23}$$

- *When system state lies between* $K-1$ *to* K

$$\mu p_K = \lambda q p_{K-1} + \mu_I (p_{-K} + p_{-K-1} + \ldots + p_{-N}) \tag{24}$$

- *When system state lies between* $N-1$ *to* N

$$\mu p_N = \lambda q p_{N-1} + \mu_I (p_{-N}) \tag{25}$$

Solving eqs. (6.20) - (6.25), we obtain

$$p_1 + p_2 + p_3 \ldots + p_N = \rho_{I_1} (p_0 + p_{-1} + p_{-2} + \ldots + p_{-N+1}) + \rho (p_1 + p_2 + \ldots + p_{N-1}) \tag{26}$$

By solving eqs. (6.23) - (6.25), we have

$$p_{-(K-1)} = \left(\frac{\rho_I}{1+\rho_I} \right)^{K-1} p_0 \quad , \text{ where } 1 \le K \le N-1$$

and $$p_{-N} = \rho_I \left(\frac{\rho_I}{1+\rho_I} \right)^{N-1} p_0 \tag{27}$$

The sum of all the probabilities is given by

$$p_0 + p_1 + p_{-1} + p_2 + p_{-2} + \ldots + p_{-N} + p_N = 1 \tag{28}$$

Now using (6.26) and (6.28), we get

$$(p_0 + p_{-1} + p_{-2} + p_{-N+1}) \left(\frac{1-\rho+\rho_{I_1}}{1-\rho} \right) + \left(1 - \frac{\rho \mu_I}{(1-\rho)\mu} \right) p_{-N} = 1 + \left(\frac{\rho^2}{1-\rho} \right) p_{N-1} \tag{29}$$

Also, $p_{N-1} = \left(\rho^{N-1} p_0 + \rho^{N-2} \rho_{I_1} p_{-1} + \rho^{N-3} \rho_{I_1} p_{-2} + \ldots + \rho^2 \rho_{I_1} p_{-N+3} + \rho_{I_1} p_{-N+2} \right)$

Then equation (6.29) becomes

$$p_0 A_1 \left[1 + \left(\frac{\rho_I}{1+\rho_I} \right) + \left(\frac{\rho_I}{1+\rho_I} \right)^2 + \ldots + \left(\frac{\rho_I}{1+\rho_I} \right)^{N-1} \right] + B_1 \left(\frac{\rho_I}{1+\rho_I} \right)^N p_0$$
$$= 1 + \left(\frac{\rho^{N+1}}{1-\rho} \right) p_0 + \left(\frac{\rho^2 \rho_{I_1}}{1-\rho} \left(\frac{\rho_I}{1+\rho_I} \right) \right) \left[\rho^{N-2} \left(\frac{\rho_I}{1+\rho_I} \right) + \rho^{N-3} \left(\frac{\rho_I}{1+\rho_I} \right)^2 + \ldots + \left(\frac{\rho_I}{1+\rho_I} \right)^{N-2} \right] p_0 \tag{30}$$

where $A_1 = \left(\frac{1-\rho+\rho_{I_1}}{1-\rho} \right)$.

Solving for p_0, we have

$$p_0 = \frac{1}{1 - \left(\frac{\rho_I}{1+\rho_I} \right)^{N-1} + \left((1-\rho)\rho_I + \rho \rho_{I_1} \right) - \rho^{N+1} \left\{ \frac{1}{1-\rho} + \frac{\rho_{I_1} \rho_I}{(1-\rho)\left(\rho + \rho_I (\rho - 1)\right)} \left(1 - \frac{1}{\rho_K} \left(\frac{\rho_I}{1+\rho_I} \right)^N \right) \right\}} \tag{31}$$

By using (6.23), we have

$$p_1 = \left[\left(\frac{\rho_I}{1+\rho_I} \right) p_0 + \left(\frac{\rho_I}{1+\rho_I} \right)^2 p_0 + \ldots + \left(\frac{\rho_I}{1+\rho_I} \right)^N p_0 \right]$$

$$= \left(\frac{\rho_I{}^2 \rho_{I_1}}{1+\rho_I} \right) \left\{ 1 - \left(\frac{\rho_I}{1+\rho_I} \right)^N \right\} p_0$$

Similarly, we write

$$p_{N-1} = \rho_{I_1} p_0 \sum_{n=1}^{N-1} \frac{\rho^{n-1} \rho_I{}^{(n+1)}}{(1+\rho_I)} \left(1 - \left(\frac{\rho_I}{(1+\rho_I)} \right)^{N-(n-1)} \right) p_0 \tag{32}$$

and $p_N = \rho_{I_1} p_0 \sum_{n=1}^{N-1} \frac{\rho^{n-1} \rho_I{}^{(n+1)}}{(1+\rho_I)} \left(1 - \left(\frac{\rho_I}{(1+\rho_I)} \right)^{N-(n-1)} \right) + \left(\frac{\rho_I{}^{N+1} \rho_{I_1}}{(1+\rho_I)^{N-1}} \right) p_0$ (33)

The steady state probabilities $P = \{ p_{-i}, p_0, p_i \,;\, i = 1, 2, 3, \ldots N \}$, can be written as

$$P_n = \begin{cases} \rho_{I_1} p_0 \sum_{n=1}^{N-1} \frac{\rho^{n-1} \rho_I{}^{(n+1)}}{(1+\rho_I)} \left(1 - \left(\frac{\rho_I}{(1+\rho_I)} \right)^{N-(n-1)} \right); & n = 1, 2, 3 \ldots N-1 \\ \rho_{I_1} p_0 \sum_{n=1}^{N-1} \frac{\rho^{n-1} \rho_I{}^{(n+1)}}{(1+\rho_I)} \left(1 - \left(\frac{\rho_I}{(1+\rho_I)} \right)^{N-(n-1)} \right) + \left(\frac{\rho_I{}^{N+1} \rho \rho_{I_1}}{(1+\rho_I)^{N-1}} \right) p_0 : & n = N \\ \left(\frac{\rho_I}{1+\rho_I} \right)^{n-1} p_0; & n = -1, -2, -3, \ldots -(N-1) \\ \rho_I \left(\frac{\rho_I}{1+\rho_I} \right)^{n-1} p_0 & n = -N \end{cases} \tag{34}$$

4. SOME PERFORMANCE MEASURES

The performance indices of the concerned queueing model can be established in terms of probabilities of the system states. Now, we formulate some performance measures of the system by using the analytical results obtained the previous section 3.

Derivation of L and L_q for infinite capacity model

- Average number of the customers present in the system is

$$L = \sum_{n=1}^{\infty} n(p_n + p_{-n}) \left[1(p_1 + p_{-1}) + 2(p_2 + p_{-2}) + 3(p_3 + p_{-3}) + \ldots \right]$$

$$= \frac{(1-\rho)}{(1-\rho+\rho_I)(1+\rho_{I_1})} \left[\sum_{n=1}^{\infty} \rho_{I_1} (1+\rho_{I_1}) + \rho_I \left(\sum_{n=2}^{\infty} (n+1) A_{I_n} \right) \right] \tag{35}$$

- Average number of the customers present in the queue is given by

$$L_q = 1(p_2 + p_{-1}) + 2(p_3 + p_{-2}) + 3(p_4 + p_{-3}) + \ldots$$

$$= \frac{(1-\rho)}{(1-\rho+\rho_I)(1+\rho_{I_1})} \left[\sum_{n=1}^{\infty} \frac{\rho_{I_1}(1+\rho_{I_1})}{(1+2\rho_{I_1})} + \rho_I \left(\sum_{n=2}^{\infty} n A_{I_n} \right) \right] \tag{36}$$

- **Average Waiting Time**

Let W_S and W_q be the average waiting time spent by a customer in the system and in the queue respectively. Using Little's formula, we obtain

(i) $W_S = \frac{L}{\lambda_{eff}}$ (37)

(ii) $W_q = \frac{L_q}{\lambda_{eff}}$, (38)

where $\lambda_{eff} = \lambda_I p_{-n} + \lambda q p_n$ and L and L_q are given in Eq. (35) and (36).

Waiting Time Distribution

Let X_q be the random variable for a period spent in the waiting line and $W_q(t)$ denotes the commulative probability distribution at any time t. Then

$$W_q(t) = prob(X_q \le t)$$

$$= W_q(0) + \sum_{n=0}^{\infty} \text{prob}\{n \text{completionsin} \le t | \text{arrivalfound}'n'\text{customersinthesystem}\}.p_n$$

where, $W_q(0) = p_0$. Thus

$$W_q(t) = \frac{1+\rho}{(1-\rho+\rho_I)(1+\rho_{I_1})} + \int_0^t \mu e^{-\mu x}\left(\frac{(\mu x)^{n-1}}{\lfloor(n-1)}(p_n)\right) + \int_0^t \mu_I e^{-\mu_I x}\left(\frac{(\mu_I x)^{n-1}}{\lfloor(n-1)}(p_{-n})\right)$$

(39)

After some algebraic manipulation, it yields

$$W_q(t) = \frac{1+\rho}{(1-\rho+\rho_I)(1+\rho_{I_1})}\left[1 + \frac{\rho_{I_1}}{1+\rho_{I_1}}\int_0^t \mu e^{-\mu x\left(\frac{1}{1+\rho_{I_1}}\right)}dx + \rho_I \int_0^t \mu_I e^{-\mu_I x}\left(\sum_{n=0}^{\infty}(\mu_I x)^n A_{I_n}\right)dx\right]$$

(40)

Remark:

In a particular case when there is no balking customer in the system i.e., $q = 1$, $\mu_I = \mu$ and $\lambda_I = \lambda$, then the proposed model coincides with the continuous service M/M/1 queueing model studied by Chew (2019).

5. ANFIS RESULTS

Using the soft computing technique namely neuro fuzzy inference system (ANFIS) which is straight forward data learning approach. In this section, we plot the system performance metrics to examine the sensitivity of (L) and (L_q) with respect to key parameters λ, μ and q. The trends of L and L_q are shown in figures 3–8. In figures 3-8, the continuous (discrete) lines without marker show the analytical results whereas continuous (discrete) lines with marker represent the ANFIS results for different values of arrival rate and service rate, respectively. In figures 3-4, we have plotted the expected system size (L) and expected queue size (L_q) by varying the service rate of real customers for various values of the balking probability q. The expected number of the customers in the system (L) and in the queue (L_q) decrease as μ increases, which is same what we expect. Figures 5 and 6 depict the trends of L and L_q corresponding to the growing values of arrival rate (λ) of the real customers for different values of q. By varying the balking probability q, figures 7-8 display the trends of W_S against the service rate and arrival rate of real consumers. It is evident that for different values of q as shown in figure 7, W_S decreases as μ increases but on the other side W_S increases as λ increases. Thus, we see that both λ and μ are the most sensitive parameters of the developed model and efforts should be made by the system designers and decision makers in choosing these parameters economically for their respective systems.

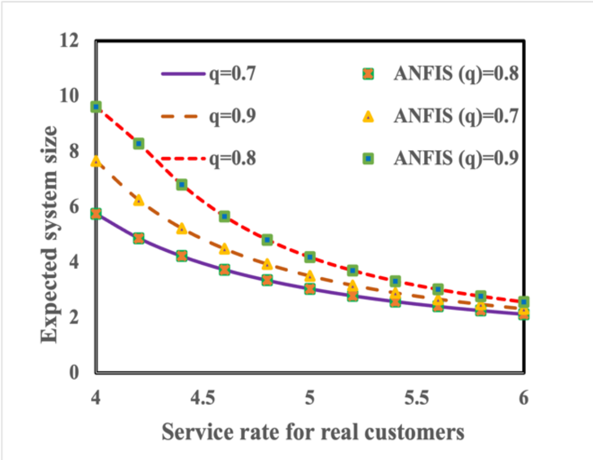

Figure 3: L vs μ for varying q .

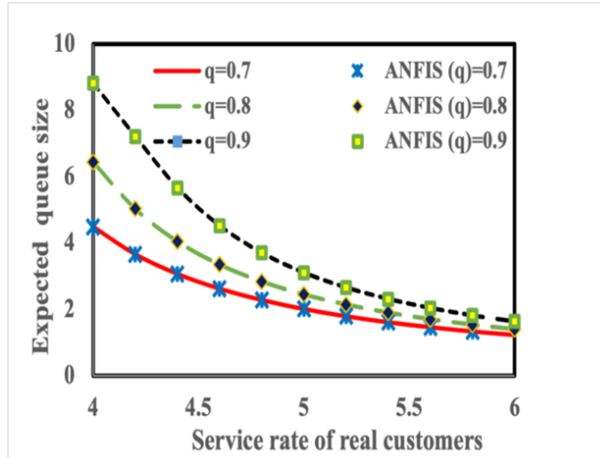

Figure 4: L_q vs μ for varying q .

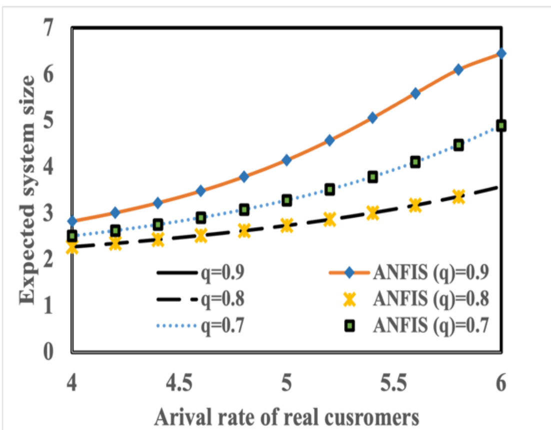

Figure 5: L vs λ for varying q .

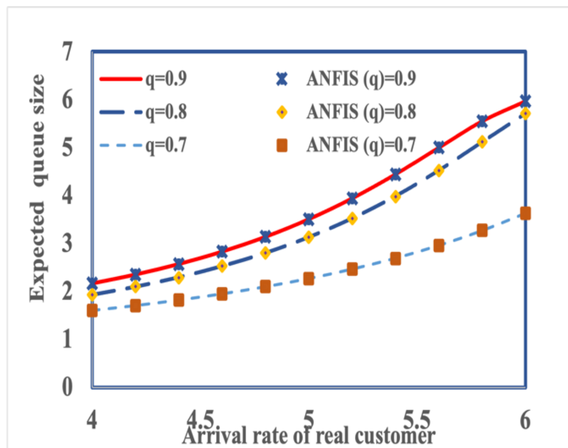

Figure 6: L_q vs λ for varying q .

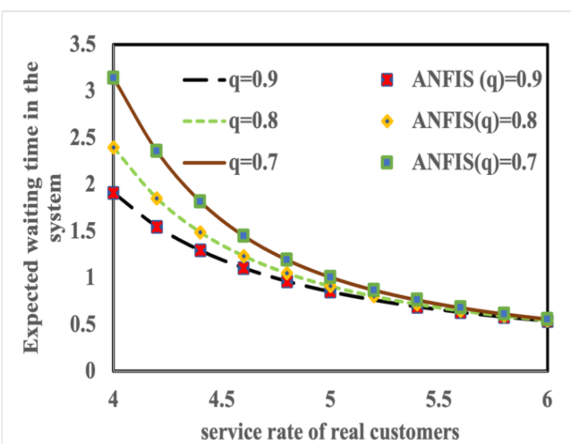

Figure 7: W_S vs μ for varying q .

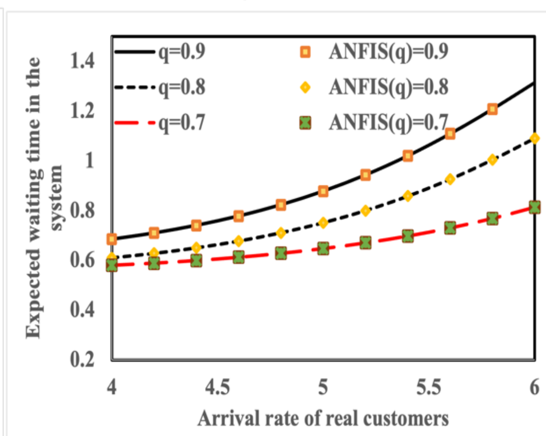

Figure 8: W_S vs λ for varying q .

6. DISCUSSION

The numerical simulation done helps us to predict the behavior of the system so as to control the arrivals of the customers. The system may also take the necessary action to prevent the longer queue. By comparing the results of the concerned problem with the standard M/M/1 queueing model, one can notice that proposed model is more realistic and quite useful to look into the problems where imaginary customers are quite common. The numerical results and the simulation may inspire to look at the future work concerned with the continuous service facility by incorporating several realistic features such as vacation, retrial orbit, bulk input, etc.

REFERENCES

Chew, S. (2019). Continuous-Service M/M/1 Queuing Systems. *Applied System Innovation 2019, Vol. 2, Page 16*, *2*(2), 16.

Harrison, P. G., & Pitel, E. (1996). The M/G/1 queue with negative customers. *Advances in Applied Probability*, *28*(2), 540–566.

Jain, M., Dhibar, S., & Sanga, S. S. (2022). Markovian working vacation queue with imperfect service, balking and retrial. *Journal of Ambient Intelligence and Humanized Computing*, *13*(4), 1907–1923.

Kumar, R., Soodan, B. S., & Sharma, S. (2021). Modelling health care queue management system facing patients' impatience using queuing theory. *Reliability: Theory and Applications*, *16*(1), 161–175.

Lai, G. R., Soh, A. C., Sarkan, H. M., Rahman, R. Z. A., & Hassan, M. K. (2015). Controlling traffic flow in multilane-isolated intersection using ANFIS approach techniques. *Journal of Engineering Science and Technology*, *10*(8), 1009–1034.

Lin, Z. C., & Liu, C. Y. (2003). Analysis and application of the adaptive neuro-fuzzy inference system in prediction of CMP machining parameters. *International Journal of Computer Applications in Technology*, *17*(2), 80–89.

López-Santana, E., Méndez-Giraldo, G., Figueroa-García, J.C. (2019). Scheduling in queueing systems and networks using ANFIS. *In: Bello, R., Falcon, R., Verdegay, J. (eds) Uncertainty Management with Fuzzy and Rough Sets. Studies in Fuzziness and Soft Computing*, Springer, *377*.

Panda, G., & Goswami, V. (2021). Equilibrium Joining Strategies of Positive Customers in a Markovian Queue with Negative Arrivals and Working Vacations. *Methodology and Computing in Applied Probability*, *24*(3), 1439–1466.

Sanga, S. S., & Jain, M. (2019). Cost optimization and ANFIS computing for admission control of M/M/1/K queue with general retrial times and discouragement. *Applied Mathematics and Computation*, *363*, 124624.

Soh, A. C., Rahman, R. Z. A., Rhung, L. G., & Sarkan, H. M. (2011). Traffic signal control based on adaptive neural-fuzzy inference system applied to intersection. *2011 IEEE Conference on Open Systems, ICOS 2011*, 231–236.

Yang, W., Probability, K. C.-J. of A., & 2001, undefined. (n.d.). A note on the GI/M/1 queue with Poisson negative arrivals. *Cambridge.Org*. Retrieved October 29, 2022.

Yang, Y., & Zhao, Q. (2012). Machine vibration prediction using ANFIS and wavelet packet decomposition. *International Journal of Modelling, Identification and Control*, *15*(3), 219–226.

Ye, Q., Methods, L. L.-C. in S.-T. and, & 2018, undefined. (2017). Analysis of MAP/M/1 queue with working breakdowns. *Taylor & Francis*, (13), 3073–3084.

Zhu, W., Patterson, B., … M. S.-I. robotics and, & 2020, undefined. (n.d.). A Markov chain model for transient analysis of handoff process in emergency departments. *Ieeexplore.Ieee.Org*. Retrieved October 28, 2022.